The New Keynesian Wage Phillips Curve: Calvo vs. Rotemberg*

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Abstract

We systematically evaluate how to translate a Calvo wage duration into an implied Rotemberg wage adjustment cost parameter in medium-scale New Keynesian DSGE models by making use of the well-known equivalence of the two setups at first order. We consider a wide range of felicity functions and show that the assumed household insurance scheme and the presence of labor taxation greatly matter for this mapping, giving rise to differences of up to one order of magnitude. Our results account for the inclusion of wage indexing, habit formation in consumption, and the presence of fixed costs in production. We also investigate the conditional and unconditional welfare implications of the wage setting schemes under efficient and distorted steady states.

JEL-Classification: E10, E30

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1 Introduction

Studying the Great Recession, economists have increasingly come to rely on nonlinear macroeconomic models, be it to study the effects of uncertainty shocks as drivers of business cycles (e.g. Fernández-Villaverde, Guerrón-Quintana, Rubio-Ramírez, and Uribe 2011; Born and Pfeifer 2014; Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez 2015) or to model the zero lower bound for the nominal interest rate (e.g. Johannsen 2014; Plante, Richter, and Throckmorton forthcoming).[1] However, the use of nonlinear solution techniques often makes it impractical to use Calvo (1983)-Yun (1996)-type nominal rigidities. First, Calvo rigidities introduce an additional state variable in the form of price/wage dispersion. Second, they give rise to meaningful heterogeneity when not embedded in the right setup (more on this below) and would require tracking distributions in the model. Rotemberg (1982)-type adjustment costs are therefore currently experiencing a renaissance.[2]

However, it is quite difficult to attach a structural interpretation to the Rotemberg adjustment cost parameter, because there is no natural equivalent in the data. In contrast, for the Calvo approach various papers have computed average price durations, e.g. Bils and Klenow (2004) and Nakamura and Steinsson (2008). The literature on *price* rigidities has therefore regularly made use of the first-order equivalence of Rotemberg- and Calvo-type adjustment frictions[3] by translating the Rotemberg adjustment costs to an implied Calvo price duration via the slope of the New Keynesian Price Phillips Curve.[4] However, such

[1]Exceptions are risk shock models like Christiano, Motto, and Rostagno (2014) and Dmitriev and Hoddenbagh (2017) where first-order approximations are sufficient.

[2]Examples of non-linear models with Rotemberg price adjustment costs include Basu and Bundick (2017), Mumtaz and Zanetti (2013), and Plante et al. (forthcoming), while Fernández-Villaverde et al. (2015), Born and Pfeifer (2017), Nath (2015), Heer, Klarl, and Maussner (2012), and Hagedorn, Manovskii, and Mitman (2018) (also) consider wage adjustment costs. Richter and Throckmorton (2016) have recently argued for using Rotemberg-type price adjustment costs to improve the model fit, not just for computational convenience.

[3]This approach can also be justified when using nonlinear methods, because the first-order approximation is only used to generate one restriction required to pin down one parameter. The equivalence, however, does not hold in case of trend inflation and incomplete indexing (see Ascari and Sbordone 2014, for a review).

[4]Early works include Roberts (1995), Keen and Wang (2007), and Nisticó (2007). This literature has also shown that the same value of the Rotemberg adjustment cost parameter can have very different economic effects, depending on the value of other structural parameters like the discount factor or the substitution elasticity.

guidance for Rotemberg *wage* adjustment costs is still missing, despite good estimates for wage durations being available for both the US (Taylor 1999; Barattieri, Basu, and Gottschalk 2014) and the euro area (Le Bihan, Montornès, and Heckel 2012). This is unfortunate, as there has recently been a renewed focus on the importance of wage rigidities (e.g. Galí 2011; Barattieri et al. 2014; Born and Pfeifer 2017).

The present study closes this gap by systematically assessing the mapping between Calvo and Rotemberg wage rigidities in a prototypical medium-scale New Keynesian model including fiscal policy.[5] A particular goal is to provide guidance for researchers working on nonlinear New Keynesian DSGE models with wage rigidities. We focus especially on how i) the other deep parameters of the model and ii) the assumed labor market structure and insurance scheme in the model affect this mapping. For example, it greatly matters whether households supply idiosyncratic labor services and insurance is conducted via state-contingent securities as in Erceg, Henderson, and Levin (2000) (EHL henceforth) or whether insurance takes place inside of a large family and a labor union supplies distinct labor services as in Schmitt-Grohé and Uribe (2006b) (SGU henceforth).[6] We also consolidate the results in the literature by providing a systematic overview of analytic expressions for the slope of the New Keynesian Wage Phillips Curve arising in the EHL setup when using different utility functions with and without consumption habits.[7]

The study most related to this part of the paper is unpublished work by Schmitt-Grohé and Uribe (2006a), who compare the slope of the New Keynesian Wage Phillips Curve arising under the EHL and the SGU setup with Calvo wage setting. However, they neither consider Rotemberg wage setting nor do they analyze the relationship to implied Calvo price durations in both setups. They also do not consider the role of fiscal policy or fixed costs in this mapping.

While the first part of the paper is concerned with the identical first-order dynamics in the Rotemberg and Calvo wage setting frameworks, at higher order the two frameworks

[5]Earlier studies like e.g. Chugh (2006, Appendix A) and Faia, Lechthaler, and Merkl (2014, footnote 29) provide the particular mapping applying in their respective model settings when calibrating the Rotemberg wage adjustment costs, but do not demonstrate how it would translate to more general models.

[6]While the former is more prominent, the latter has been used e.g. in Trigari (2009), Pariés, Sørensen, and Rodriguez-Palenzuela (2011), and Born, Peter, and Pfeifer (2013).

[7]An early precursor of this work is Sbordone (2006).

generally differ. In the second part of the paper we investigate some of these differences by analyzing the welfare implications of different types of wage rigidities in a business cycle context. We theoretically show that if the steady state is efficient, welfare conditional on zero initial wage dispersion is identical under Calvo and Rotemberg wage setting. In contrast, from an unconditional welfare perspective the welfare losses under Calvo wage setting are bigger. We also show numerically that if the steady state is not efficient, then Calvo wage setting tends to generate larger welfare losses. These results mirror the findings of Nisticó (2007), Lombardo and Vestin (2008), and Damjanovic and Nolan (2011) for the case of price stickiness. We also investigate the welfare differences arising between the EHL and SGU wage setting scheme.

The paper proceeds as follows. Sections 2 and 3 consider the EHL and SGU setups, respectively. Section 4 provides a numerical comparison. Section 5 performs the welfare analysis. Section 6 concludes. An appendix with detailed derivations and accompanying computer codes is available online.

2 New Keynesian Phillips Curve in the EHL-setup

In this section we lay out the respective prototypical household setups used in EHL and then derive the slope of the New Keynesian Wage Phillips Curve under Calvo and Rotemberg pricing. In the background, but not of interest here, there are a continuum of firms producing differentiated intermediate goods and a final good firm bundling intermediate goods to a final good. In addition, there is a fiscal authority that finances government spending with distortionary labor and consumption taxation and transfers and a monetary authority conducting monetary policy, e.g. according to a Taylor-type interest rate rule.

2.1 Setup

Following EHL, we assume that the economy is populated by a continuum of monopolistically competitive, infinitely-lived households $j, j \in [0, 1]$, supplying differentiated labor services N_t^j at wage W_t^j to intermediate goods producers who aggregate them into a composite labor input N_t^d with cost W_t using a Dixit and Stiglitz (1977) aggregator.

2.2 Calvo wage setting

In case of Calvo pricing, the household is not able to readjust its wage in any given period with probability θ_w. Therefore, it chooses today's optimal wage W_t^* to maximize the expected utility over the states of the world where this wage is operative:

$$\max_{W_t^*} V_t = E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k U\left(C_{t+k|t}^j, N_{t+k|t}^j, \cdot\right) , \tag{2.1}$$

where V is the utility function and U is the felicity function with partial derivatives $U_C > 0$ and $U_N < 0$. The dot denotes additional variables potentially entering the felicity function (e.g. lagged consumption in the case of habits), and where $0 < \beta < 1$ is the (growth-adjusted) discount factor. The subscript $t+k|t$ indicates a variable in period $t+k$ conditional on having last reset the wage at time t. When choosing the optimal wage W_t^*, the household does so taking into account the demand for its labor services

$$N_{t+k|t}^j = \left(\frac{W_{t+k|t}^j}{W_{t+k}}\right)^{-\varepsilon_w} N_{t+k}^d , \tag{2.2}$$

where the wage operative in period $t+k$, $W_{t+k|t}^j$, is given by the originally chosen wage W_t^* times a term $\Gamma_{t,t+k}^{ind}$ that reflects the indexing of wages to (past) inflation:

$$W_{t+k|t}^j = \Gamma_{t,t+k}^{ind} W_t^* . \tag{2.3}$$

We keep this term generic to encompass the varying indexing schemes in the literature and only require that there is full indexing in steady state, i.e. $\Gamma_k^{ind} = \Pi^k$.[8] Note that $\Gamma_{t,t}^{ind} = 1$. The final constraint of this problem is the budget constraint

$$(1+\tau_{t+k}^c)P_{t+k}C_{t+k|t}^j = (1-\tau_{t+k}^n)W_{t+k|t}^j N_{t+k|t}^j + X_t , \tag{2.4}$$

[8] Our formulation encompasses, e.g., the partial indexation scheme of Smets and Wouters (2007), which is of the form $\Gamma_{t,t+k}^{ind} = \prod_{s=1}^{k} \Pi_{t+s-1}^{\iota}\Pi^{1-\iota}$, where $\iota \in [0,1]$ denotes the degree of indexing to past inflation and Π without subscript denotes steady state inflation. Another indexing scheme nested is the one by Christiano, Eichenbaum, and Evans (2005), who use full indexation to past inflation with $\iota = 1$. The absence of indexing is characterized by $\iota = 0$ and $\Pi = 1$ so that $\Gamma_{t,t+k}^{ind} = 1 \,\forall\, k$.

where the household earns income from supplying differentiated labor N_t^j at the nominal wage rate W_t^j, which is taxed or subsidized at rate τ_t^n, and spends its income on consumption C_t^j, priced at the price of the final good P_t and taxed at rate τ_t^c. In this budget constraint all additive terms that drop from the current optimization problem when taking the derivative with respect to W_t^* (e.g. capital income or transfers) have been lumped together in X_t.

Define the after-tax marginal rate of substitution as

$$MRS_{t+k|t} = -\frac{\left(1+\tau_{t+k}^c\right)}{\left(1-\tau_{t+k}^n\right)} \frac{U_{N,t+k|t}}{V_{C,t+k|t}} , \tag{2.5}$$

where subscripts C and N denote partial derivatives and the index j has been suppressed.[9] The well-known optimality condition for the optimal wage W_t^* prescribes that households set a desired markup over the weighted average of expected future marginal rates of substitution. In its log-linearized version it yields

$$\hat{W}_t^* = (1-\beta\theta_w) \sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \left[\widehat{MRS}_{t+k|t} + \hat{P}_{t+k} - \hat{\Gamma}_{t,t+k}^{ind}\right] , \tag{2.6}$$

where hats denote percentage deviations from steady state. In order to derive the New Keynesian Wage Phillips Curve, one needs to express the previous equation recursively and aggregate over households j. Aggregation in particular implies replacing the idiosyncratic marginal rate of substitution $\widehat{MRS}_{t+k|t}$ by an expression not depending on the initial period in which household j last reset the wage.

Log-linearizing (2.5) around the deterministic steady state (denoted with omitted time indices), and combining it with the assumption of complete markets and equal initial wealth, yields

$$\widehat{MRS}_{t+k|t} = \widehat{MRS}_{t+k} + \underbrace{\left[-\frac{V_{CN} \times N}{V_{CC} \times C} \varepsilon_c^{mrs} + \varepsilon_n^{mrs}\right]}_{\equiv \varepsilon_{tot}^{mrs}} \left(\hat{N}_{t+k|t} - \hat{N}_{t+k}\right) , \tag{2.7}$$

where ε_n^{mrs} and ε_c^{mrs} denote the steady state elasticities of the marginal rate of substitution with respect to labor and consumption, respectively, and ε_{tot}^{mrs} is the total elasticity of the

[9] This formulation allows for non-time separable utility in consumption as introduced by habits, but excludes habits in leisure (e.g. Uhlig 2007).

MRS. The latter simplifies to ε_n^{mrs} in the case of additively separable preferences as in Erceg et al. (2000), because $V_{CN} = 0$. $\widehat{MRS}_{t+k}$ is the average MRS in the economy.

Using equation (2.7), the linearized law of motion for the aggregate wage level and defining wage inflation $\Pi_{w,t} = \frac{W_t}{W_{t-1}}$, the New Keynesian Wage Phillips Curve follows after some tedious algebra as

$$\hat{\Pi}_t^w = \beta E_t \hat{\Pi}_{w,t+1} - \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\varepsilon_w \varepsilon_{tot}^{mrs})}\hat{\mu}_t^w - \frac{\beta\theta_w}{1-\theta_w} E_t \hat{\Gamma}_{t,t+1}^{ind} + \frac{\theta_w}{1-\theta_w}\hat{\Gamma}_{t-1,t}^{ind} \ , \tag{2.8}$$

where $\hat{\mu}_t^w$ defines the deviation of the wedge between the average marginal rate of substitution and the real wage from its long-run value, i.e. the steady state markup:

$$\hat{\mu}_t^w \equiv \left(\hat{W}_t - \hat{P}_t\right) - \widehat{MRS}_t \ . \tag{2.9}$$

Equation (2.8) has the familiar intuition that if $\hat{\mu}_t^w < 0$, the wage markup is below its long-run value, inducing wage setters ceteris paribus to adjust wages upwards, leading to wage inflation.

2.3 Rotemberg wage setting

In case of Rotemberg pricing the problem of household j is choosing W_t^j to maximize

$$V_t = E_t \sum_{k=0}^{\infty} \beta^k U\left(C_{t+k}^j, N_{t+k}^j, \cdot\right) \ , \tag{2.10}$$

taking into account the demand for its labor variety

$$N_{t+k}^j = \left(\frac{W_{t+k}^j}{W_{t+k}}\right)^{-\varepsilon_w} N_{t+k}^d \tag{2.11}$$

and subject to the budget constraint

$$(1+\tau_{t+k}^c)P_{t+k}C_{t+k}^j = (1-\tau_{t+k}^n)W_{t+k}^j N_{t+k}^j - \frac{\phi_w}{2}\left(\frac{1}{\Gamma_{t+k-1,t+k}^{ind}}\frac{W_{t+k}^j}{W_{t+k-1}^j} - 1\right)^2 \Xi_{t+k} + X_{t+k} \ . \tag{2.12}$$

Here, the second-to-last term represents the quadratic Rotemberg costs of adjusting the wage, with ϕ_w being the Rotemberg wage adjustment cost parameter. The costs are proportional to the nominal adjustment cost base Ξ_{t+k} and arise whenever wage changes differ from the indexed inflation rate $\Gamma^{ind}_{t+k-1,t+k}$.[10] X_{t+k} again captures additive terms not related to the current optimization problem. After imposing symmetry and making use of the definition of the after-tax MRS, equation (2.5), the resulting FOC can be written as

$$0 = \varepsilon_w \frac{MRS_t}{\frac{W_t}{P_t}}(1-\tau^n_t) + \left\{ (1-\varepsilon_w)(1-\tau^n_t) - \phi_w \left(\frac{1}{\Gamma^{ind}_{t-1,t}} \Pi_{w,t} - 1 \right) \Pi_t \frac{1}{N_t} \frac{1}{\Gamma^{ind}_{t-1,t}} \frac{\frac{\Xi_t}{P_t}}{\frac{W_{t-1}}{P_{t-1}}} \right\}$$
$$+ E_t \beta \frac{V_{C,t+1}}{V_{C,t}} \frac{(1+\tau^c_t)}{(1+\tau^c_{t+1})} \frac{1}{N_t} \frac{1}{\frac{W_t}{P_t}} \left\{ \phi_w \left(\frac{1}{\Gamma^{ind}_{t,t+1}} \Pi_{w,t+1} - 1 \right) \frac{1}{\Gamma^{ind}_{t,t+1}} \Pi_{w,t+1} \frac{\Xi_{t+1}}{P_{t+1}} \right\} . \tag{2.13}$$

Linearizing (2.13) around the steady state and making use of the definition of $\hat{\mu}^w_t$, (2.9), yields

$$\hat{\Pi}_{w,t} = \beta E_t \hat{\Pi}_{w,t+1} - \frac{(\varepsilon_w - 1)(1-\tau^n)\aleph}{\phi_w} \hat{\mu}^w_t \ , \tag{2.14}$$

where $\aleph \equiv \frac{N \times W}{\Xi}$ denotes the steady state share of the wage bill in the adjustment cost base.[11] It is notable that despite allowing for indexing the Rotemberg framework does not give rise to a hybrid wage Phillips Curve with a backward-looking inflation term. The reason is that all firms in the Rotemberg wage adjustment framework adjust their wage each period. In contrast, in the Calvo framework, there are non-adjusters whose wage evolves according to the indexing process, which gives rise to a backward-looking term. Most papers assume that wage adjustment costs are proportional to either current or steady state output.[12] Thus, the real steady state adjustment costs base Ξ/P is equal to output Y, which is produced via a production function of the type $Y = F(K,N) - \Phi$, where F is a constant returns to scale production function and $\Phi \geq 0$ denotes fixed costs in production. The literature typically either abstracts from fixed costs, i.e. $\Phi = 0$, or sets them to the value of monopolistic pure profits so that there is no incentive for entry or exit in steady state. In that case, $\Phi = \varepsilon_p^{-1} Y$, where $\varepsilon_p > 0$ is the elasticity of substitution between monopolistically competitive intermediate goods firms. Steady state output then

[10] Papers typically assume full indexing with the steady state interest rate, i.e. $\Gamma^{ind}_{t+k-1,t+k} = \Pi$. Some papers like Fernández-Villaverde et al. (2015) use the formulation $\tilde{\phi}_w/2 \times (W^j_{t+k}/W^j_{t+k-1} - \Pi)^2$. Our specification is equivalent to this formulation with $\phi_w = \tilde{\phi}_w \Pi^2$.

[11] Note that while we allow tax rates to vary, tax rate changes only have a direct effect on the Wage Phillips Curve via their effect on the after-tax marginal rate of substitution.

[12] For the purpose of this paper it is only important that this term is exogenous from the perspective of the wage setting household so that the effects of household decisions on it are not internalized.

is $Y = \frac{\varepsilon_p - 1}{\varepsilon_p} F(K, N)$.

With firms choosing a gross markup of $\varepsilon_p / (\varepsilon_p - 1)$ over marginal cost, $\aleph$ is given by

$$\aleph = \frac{WN}{\Xi} = \frac{\frac{\varepsilon_p - 1}{\varepsilon_p} F_N N}{Y} = \begin{cases} \frac{\varepsilon_p - 1}{\varepsilon_p} (1 - \alpha), & \text{if } \Phi = 0, \\ (1 - \alpha), & \text{if } \Phi = \varepsilon_p^{-1} Y. \end{cases} \tag{2.15}$$

Here, $1 - \alpha$ denotes the steady state elasticity of the production function with respect to labor, e.g. the labor exponent in a Cobb-Douglas production function. Expression (2.15) shows that the relevant steady state labor share $\aleph$ is bigger in case of fixed costs, because net output Y in the denominator only includes capital and labor payments, while in case of no fixed costs, it also includes pure profits. Hence, the slope of the Wage Phillips Curve is ceteris paribus flatter in the absence of fixed costs.

2.4 Mapping Calvo durations to Rotemberg adjustment costs

Comparing the slopes of the two Wage Phillips Curves, equations (2.8) and (2.14), yields

$$\frac{(1 - \theta_w)(1 - \beta \theta_w)}{\theta_w (1 + \varepsilon_w \varepsilon_{tot}^{mrs})} = \frac{(\varepsilon_w - 1)(1 - \tau^n)\aleph}{\phi_w}, \tag{2.16}$$

from which the Rotemberg wage adjustment cost parameter ϕ_w^{EHL} in the EHL framework implied by a particular Calvo wage duration θ_w can be inferred as

$$\phi_w^{EHL} = \frac{(\varepsilon_w - 1)(1 - \tau^n)\aleph}{(1 - \theta_w)(1 - \beta \theta_w)} \theta_w (1 + \varepsilon_w \varepsilon_{tot}^{mrs}) . \tag{2.17}$$

The left-hand side of equation (2.16) shows that, similar to the case of the New Keynesian Price Phillips curve, the discount factor β and the Calvo wage duration θ_w determine the slope of the Wage Phillips Curve in the Calvo case. But there is an additional correction factor in the denominator of equation (2.16) that is a function of the elasticity of substitution ε_w and the total elasticity of the marginal rate of substitution, ε_{tot}^{mrs}. This correction factor arises from the EHL setup in the Calvo case due to the idiosyncratic marginal rate of substitution being used to evaluate the labor-leisure trade-off when deciding on the new wage, while the New Keynesian Wage Phillips Curve is written in terms of the average marginal rate of substitution. As equation (2.7) shows, the idiosyncratic MRS of a wage re-setter is equal to the average MRS plus a correction factor accounting for differences in hours worked relative to the average. These differences in hours worked, in turn, arise from the reset wage W_t^* differing from the aggregate one (see the labor demand equation (2.2)). Consequently, this additional correction factor drops out when either the total elasticity of the MRS ε_{tot}^{mrs} is zero, or the substitution elasticity ε_w is zero (or both). Both these cases result in the idiosyncratic MRS being equal to the average one. In the first case, the idiosyncratic MRS is completely unresponsive to differences in hours worked arising from wage stickiness. In the second case, the j labor services are perfect complements so that even arbitrarily large wage differences do not translate to any differences in hours worked.

Table 1 displays the respective expressions for ε_{tot}^{mrs} for different felicity functions (see Appendix C for details). In case of standard additively separable preferences and for Greenwood, Hercowitz, and Huffman (1988)-preferences, ε_{tot}^{mrs} simply corresponds to the inverse Frisch-elasticity parameter φ. For additively separable preferences with log leisure, the total elasticity is pinned down by the ratio of hours worked to leisure. For multiplicatively separable Cobb-Douglas-type preferences, ε_{tot}^{mrs} depends on the degree of risk aversion, the weight of leisure in the utility function, and the ratio of hours worked to leisure.

With Frisch elasticity estimates ranging from 0.75 using micro data (Chetty, Guren, Manoli, and Weber 2011) to 2-4 using macro data (e.g. Smets and Wouters 2007; King and Rebelo 1999) as well as a share of hours worked in total time of 0.2 to 0.33, plausible values for the elasticity range between 0.25 and 1.5. With multiplicative preferences, realistic calibrations are in the same range as those obtained for separable preferences. For example, Backus, Kehoe, and Kydland (1992) use $\sigma = 2$, $\eta = 0.34$, and $N/(1-N) = 0.5$ so that $\varepsilon_{tot}^{mrs} \approx 0.75$.[13]

For the Rotemberg case on the right-hand side of (2.16), the slope depends on the elasticity of substitution, the Rotemberg adjustment cost parameter ϕ_w, and on the share of the wage bill in the adjustment cost tax base $\aleph$. In contrast to the previously considered

[13] The lower bound is obtained with $\varepsilon_{tot}^{mrs} = 0$ for $\sigma = 0$, reaches $\varepsilon_{tot}^{mrs} = N/(1-N) = 0.5$ for $\sigma = 1$ (i.e. the additively separable case) and then keeps increasing.

Table 1: Elasticity ε_{tot}^{mrs} for different felicity functions

	$U(C,N)$	ε_{tot}^{mrs}	Habits
Add. separable	$\dfrac{C^{1-\sigma}-1}{1-\sigma} - \psi \frac{N^{1+\varphi}}{1+\varphi}$	φ	✓
GHH (1988)	$\dfrac{\left(C-\psi N^{1+\varphi}\right)^{1-\sigma}-1}{1-\sigma}$	φ	✓
Add. sep., log leisure	$\dfrac{C^{1-\sigma}-1}{1-\sigma} + \psi \log\left(1-N\right)$	$\dfrac{N}{1-N}$	✓
Multipl. separable	$\dfrac{\left(C^{\eta}\left(1-N\right)^{1-\eta}\right)^{1-\sigma}-1}{1-\sigma}$	$\left[1-\dfrac{\left(1-\eta\right)\left(\sigma-1\right)}{\eta(1-\sigma)-1}\right] \times \dfrac{N}{1-N}$	✓(*)

Notes: Total elasticity of the after-tax marginal rate of substitution, ε_{tot}^{mrs}, for additively separable preferences in consumption and hours worked (first row), for Greenwood, Hercowitz, and Huffman (1988)-type preferences (second row), additively separable preferences in consumption and log leisure (third row), and multiplicative preferences (fourth row). The last column indicates whether the computed elasticity is robust to the inclusion of internal or external habits in consumption of the form $C_t - \phi_c C_{t-1}$. (*) For multiplicatively separable preferences, the resulting expression becomes somewhat more complex, see Appendix C.1.2.

Calvo case, the slope of the Wage Phillips Curve with Rotemberg pricing is decreasing in the labor tax rate τ^n. The reason is that the labor tax rate drives a wedge between the real wage and the marginal rate of substitution. In the limit case of $\tau^n \to 1$, it does not pay off for the household to invest any resources into changing the nominal wage. Wage inflation then becomes completely decoupled from $\hat{\mu}_t$.[14]

Two remarks are in order. The first, technical one, is that Rotemberg wage adjustment cost estimates from papers abstracting from labor taxes cannot be translated directly to models with such taxes, because they will correspond to a flatter Phillips curve than intended. The second point is an economic one. If one believes that the Rotemberg price adjustment cost parameter is structural, then equation (2.14) implies that permanent increases in labor taxes can flatten the Wage Phillips Curve. Therefore, if presumed permanent, the gradual increase of labor taxes in the U.S. from below 15% before 1960 to its new plateau of about 23% is, ceteris paribus, associated with a flattening of the Wage

[14] The same does not hold true for the time-dependent Calvo wage adjustment. Whenever the household is allowed to reset its wage, it can do so costlessly. For that reason, as shown in (2.17), the wage adjustment cost parameter ϕ_w implied by a particular Calvo duration, which appears in the denominator of (2.14), is decreasing at rate $(1-\tau^n)$, canceling the overall effect of τ^n.

Phillips Curve of 8 percentage points in this framework.[15]

3 New Keynesian Phillips Curve in the SGU-setup

In this section we first derive the slope of the New Keynesian Wage Phillips curve in the Schmitt-Grohé and Uribe (2006b)-setup under Calvo pricing and under Rotemberg pricing and then map them into each other.

3.1 Setup

Schmitt-Grohé and Uribe (2006b) assume that the economy is populated by a household with a continuum of members that supply the same homogenous labor service N_t, have the same consumption level due to insurance within the household, and work the same amount of hours. This contrasts with EHL, where households supply differentiated labor services and insurance takes place via complete markets.[16] The homogenous labor input in the SGU setup is supplied to a labor union that takes its members utility into account and acts as a monopoly supplier of a continuum of j differentiated labor services N_t^j. These differentiated labor services are bundled into a composite labor input by intermediate goods producers exactly as in the EHL setup in section 2.

The household has lifetime utility function

$$V_t = E_t \sum_{k=0}^{\infty} \beta^k U(C_{t+k}, N_{t+k}, \cdot) \ , \tag{3.1}$$

where $N_{t+k} = \int_0^1 N_{t+k}^j \, dj$ is the market clearing condition assuring that total hours worked across all markets equal the supply by households. The household's nominal budget

[15]Appendix A.2 shows that the coefficient on labor tax rate *changes* in the linearized wage PC in the Rotemberg case is always 0. Thus, under both the Calvo and the Rotemberg case, the direct effect of changes in tax rates on the wage Phillips Curve only comes via its effect on the after-tax MRS and is therefore identical under both setups.

[16]Galí (2015) provides a different microfoundation of the EHL setup. He assumes a household with j members, each supplying a differentiated labor service, who are perfectly insured within the family. He then pairs this with j labor unions responsible for the wage setting in market j. Because unions only take the utility of their members into account, i.e. use the idiosyncratic MRS, this setup isomorphic to EHL.

constraint is

$$(1+\tau^c_{t+k})P_{t+k}C_{t+k} \leq (1-\tau^n_{t+k})\int_0^1 W^j_{t+k}N^j_{t+k}\,dj + X_{t+k}\ , \tag{3.2}$$

where the household earns income from differentiated labor N^j_{t+k} at the nominal wage rate W^j_{t+k} through the labor services supplied by the union and X_{t+k} again captures unrelated additive terms.

3.2 Calvo wage setting

The labor union chooses the optimal wage W^*_t in all labor markets where it is able to reoptimize in order to maximize its members' utility, equation (3.1). It takes into account the demand for labor variety j, equation (2.2), and the relevant part of the budget constraint (3.2):

$$(1+\tau^c_{t+k})P_{t+k}C_{t+k} = \left(1-\tau^n_{t+k}\right)W^{\varepsilon_w}_{t+k}N^d_{t+k}\theta^k_w\left(\Gamma^{ind}_{t,t+k}W^*_t\right)^{1-\varepsilon_w}\quad . \tag{3.3}$$

The latter makes use of the fact that, at each point in time $t+k$, the union is able to reset the wage in a fraction $1-\theta_w$ of labor markets, which therefore become irrelevant for the wage setting decision at time t. This leaves a fraction θ^k_w of labor markets where the time t optimal wage W^*_t is still active. Taking the FOC, the New Keynesian Wage Phillips Curve follows after some tedious algebra as

$$\hat{\Pi}^w_t = \beta E_t\hat{\Pi}^w_{t+1} - \frac{(1-\beta\theta_w)(1-\theta_w)}{\theta_w}\hat{\mu}^w_t - \frac{\beta\theta_w}{1-\theta_w}E_t\hat{\Gamma}^{ind}_{t,t+1} + \frac{\theta_w}{1-\theta_w}\hat{\Gamma}^{ind}_{t-1,t}\ . \tag{3.4}$$

Comparing the slope of the Wage Phillips Curve in (3.4) to the one of EHL in (2.8), the EHL slope is smaller by a factor of $(1+\varepsilon_w\varepsilon^{mrs}_{tot})^{-1}$. The reason is that in the SGU setup, due to the family structure where everyone consumes the same and works the same hours, wage re-setters use the average MRS to evaluate the labor leisure trade-off, not the idiosyncratic one. Hence, no correction factor is needed.

3.3 Rotemberg wage setting

The Rotemberg problem of the labor union is similar to the household wage setting problem in the EHL case. The relevant part of the budget constraint is given by

$$(1+\tau_t^c)P_tC_t = (1-\tau_t^n)\int_0^1 W_t^j N_t^j\ dj - \frac{\phi_w}{2}\int_0^1 \left(\frac{1}{\Gamma_{t-1,t}^{ind}}\frac{W_t^j}{W_{t-1}^j} - 1\right)^2\ dj\ \Xi_t\ . \tag{3.5}$$

Following the steps outlined in section 2.3, it can be verified that this leads to the same Wage Phillips Curve as in the EHL case:

$$\hat{\Pi}_{w,t} = \beta E_t \hat{\Pi}_{w,t+1} - \frac{(\varepsilon_w - 1)(1-\tau^n)\aleph}{\phi_w}\hat{\mu}_t^w\ . \tag{2.14}$$

3.4 Mapping Calvo durations to Rotemberg adjustment costs

Comparison of the slopes of the two Wage Phillips Curves, equations (3.4) and (2.14), yields an expression for the Rotemberg parameter ϕ_w implied by a Calvo wage duration θ_w in the SGU setup:

$$\phi_w^{SGU} = \frac{(\varepsilon_w - 1)(1-\tau^n)\aleph}{(1-\theta_w)(1-\beta\theta_w)}\theta_w\ , \tag{3.6}$$

which differs from the EHL case, equation (2.17). The latter has an additional term $(1+\varepsilon_w\varepsilon_{tot}^{mrs})$ arising from the idiosyncratic MRS being used to evaluate the labor leisure trade-off when deciding on the new optimal wage instead of the aggregate one.

4 Comparison of EHL- vs. SGU-style insurance schemes

4.1 Rotemberg wage adjustment costs

Table 2 shows the implied Rotemberg wage adjustment cost parameter corresponding to an implied Calvo wage duration of 4 quarters ($\theta_w = 0.75$) for different parameter values in the SGU and EHL frameworks at quarterly frequency. All parameters except for the one under consideration are kept at their baseline values. For the baseline calibration we choose a discount factor of $\beta = 0.99$, corresponding to a 4% real interest rate. The labor tax rate is set to 0.21, which is the mean U.S. effective tax rate over the sample

Table 2: Implied Rotemberg adjustment cost parameters ϕ_w (quarterly model)

	$\varepsilon^{mrs}_{tot} = 0.25$	$\varepsilon^{mrs}_{tot} = 1$	$\varepsilon^{mrs}_{tot} = 1.5$	$\beta = 0.985$	$\beta = 0.99$	$\beta = 0.995$
SGU	61.36	61.36	61.36	60.48	61.36	62.27
EHL	230.10	736.31	1073.79	725.74	736.31	747.19
	$\varepsilon_w = 6$	$\varepsilon_w = 11$	$\varepsilon_w = 21$	$\tau^n = 0$	$\tau^n = 0.21$	$\tau^n = 0.4$
SGU	30.68	61.36	122.72	77.67	61.36	46.60
EHL	214.76	736.31	2699.81	932.04	736.31	559.22
	$\Phi = \varepsilon_w^{-1} Y$	$\Phi = 0$				
SGU	61.36	55.78				
EHL	736.31	669.37				

Notes: Implied Rotemberg wage adjustment cost parameter ϕ_w that corresponds to an implied Calvo wage duration of 4 quarters ($\theta_w = 0.75$) for different parameter values in the SGU and EHL framework. All other parameters are kept at their baseline value: $\beta = 0.99$, $\tau^n = 0.21$, $\varepsilon_w = \varepsilon_p = 11$, $\alpha = 0.3$, $\varepsilon^{mrs}_{tot} = 1$, $\Phi = \varepsilon_w^{-1} Y$.

1960Q1:2015Q4, computed following Jones (2002). The substitution elasticities are set to $\varepsilon_w = \varepsilon_p = 11$, implying a steady state markup of 10%. $\aleph$ is set to 2/3, corresponding to an exponent of capital in a Cobb-Douglas production function of $\alpha = 0.3$ and the presence of fixed costs that make steady state firm profits 0. The total elasticity of the marginal rate of substitution, ε^{mrs}_{tot}, is set to 1 as is the case with additively separable preferences and an inverse Frisch elasticity of $\varphi = 1$.

As can be seen in the rows labeled SGU and EHL, the particular household setup assumed makes a big difference due to the multiplicative $(1 + \varepsilon_w \varepsilon^{mrs}_{tot})$ factor appearing in the EHL-setup. For our baseline parameterization, this factor amounts to $1 + 11 \times 1 = 12$. This factor is also what makes the slope of the Wage Phillips Curve increase (almost) proportionally with the total elasticity of the marginal rate of substitution, ε^{mrs}_{tot}, in the EHL-setup (second row, left panel). In contrast, ε^{mrs}_{tot} does not affect the slope in the SGU case (first row, left panel). The implied Rotemberg parameter increases proportionally in the elasticity of substitution between goods ε_w for the SGU setup (third row, left panel). However, it increases overproportionally in the EHL setup (fourth row, left panel). Increasing ε_w by a factor of 3.5 from 6 to 21 results in an increase of the implied ϕ_w by a factor of 12.6. Assuming the absence of fixed costs, $\Phi = 0$, hardly changes the implied cost parameter in both setups for plausible calibrations (fifth and sixth rows, left panel). The first two rows of the right panel of Table 2 show that the effect of varying the discount factor β is relatively minor in both setups. Finally, the third and fourth rows of the right panel show that the steady state labor tax rate τ^n significantly impacts the implied Rotemberg costs parameter as already discussed in section 2.4.

4.2 Business-cycle dynamics under Calvo wage setting: monetary policy shock example

To gauge the economic significance of the difference in the Wage Phillips Curve implied by the SGU- versus EHL-style insurance schemes under Calvo wage setting, we explore the impulse response functions (IRFs) to a monetary policy shock in the quarterly benchmark New Keynesian model with sticky prices and wages à la Calvo outlined in Galí (2015, Chapter 6). We deliberately keep the exposition at a minimum and refer to the textbook chapter for details.

The intermediate goods of firm $i \in [0,1]$ are produced using the production function

$$Y_t^i = A_t \left(N_t^i\right)^{1-\alpha} \quad , \tag{4.1}$$

with $0 < \alpha < 1$ measuring the decreasing returns to scale, A_t being an $AR(1)$ exogenous technology shock process with mean 1, and N_t^i a Dixit-Stiglitz aggregate of differentiated labor services $N_t^{i,j}$, $j \in [0,1]$ with substitution elasticity ϵ_w. There are no fixed costs of production. The final good is a Dixit-Stiglitz aggregate of the intermediate goods with substitution elasticity ϵ_p.

Household member $j \in [0,1]$ has the felicity function

$$U_t^j = Z_t \left(\frac{\left(C_t^j\right)^{1-\sigma} - 1}{1-\sigma} - \frac{\left(N_t^j\right)^{1+\varphi}}{1+\varphi} \right) \quad , \tag{4.2}$$

where σ is the risk aversion parameter and φ is the inverse Frisch elasticity. Z_t is an $AR(1)$ exogenous demand shock process with mean 1. Due to the assumption of risk sharing via complete markets (EHL) or within the large family (SGU), the consumption level is the same for all j, i.e. $C_t^j = C_t \,\forall\, j$. In addition, in the SGU case, all household members supply the same labor service to a labor union, so that $N_t^j = N_t \, \forall \, j$.

Monetary policy is conducted using a Taylor rule of the form

$$R_t = \frac{1}{\beta} \left(\frac{\Pi_t^p}{\Pi^p} \right)^{\phi_\pi} \left(\frac{Y_t}{Y} \right)^{\phi_y} e^{\nu_t} \ , \tag{4.3}$$

where ν_t is a mean zero $AR(1)$ exogenous monetary policy shock process, and Π^p and Y denote steady-state price inflation and outout, respectively. The calibration is summarized in Table 3. The model is solved using first-order perturbation techniques.

Table 3: Parameter Values

Parameter	**Value**	**Description**
α	0.250	capital share
β	0.990	discount factor
σ	1.000	inverse elasticity of intertemporal substitution
φ	5.000	inverse Frisch elasticity
ϕ_π	1.500	inflation feedback Taylor Rule
ϕ_y	0.125	output feedback Taylor Rule
ϵ_p	9.000	substitution elasticity intermediate goods
θ_p	0.750	Calvo parameter price setting
ϵ_w	4.500	substitution elasticity labor services
θ_w	0.750	Calvo parameter wage setting
Π^p	1	Steady state gross price inflation
ρ_a	0.900	autocorrelation technology shock
ρ_ν	0.500	autocorrelation monetary policy shock
ρ_z	0.500	autocorrelation demand shock

Figure 1 displays the results of a 1 percentage point (annualized) monetary policy shock. The blue solid line shows the IRFs from the Galí (2015, Chapter 6.2.1) baseline EHL-type model. The red dashed line displays the IRFs from the SGU-type model with the same Calvo wage stickiness parameter, which implies a steeper wage Phillips Curve. As a consequence, output movements are very similar in both setups, but the responses of wage and price inflation as well as the real wage are stronger.

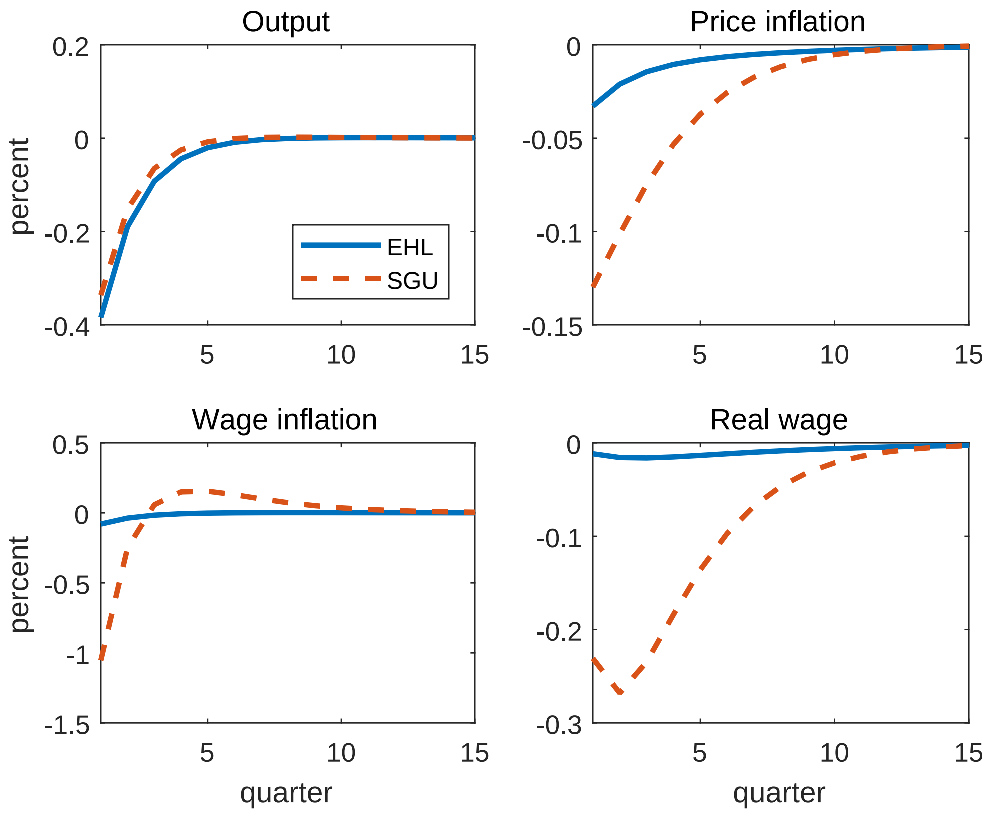

Figure 1: Impulse response functions to 1 percentage point (annualized) monetary policy shock under Calvo at time 1. *Note:* blue solid line: Galí (2015, Chapter 6.2.1) EHL-type model; red dashed line: same model but with SGU-style insurance scheme. IRFs are in percent.

5 Welfare implications

Calvo and Rotemberg price and wage setting are identical up to first order in the absence of trend inflation.[17] However, the two price/wage setting models differ at higher order, potentially giving rise to different welfare implications as welfare computations generally require at least a second-order approximation (see e.g. Woodford 1999).

For the price stickiness case, Nisticó (2007) has shown that, conditional on initial price dispersion being zero, welfare up to second order is identical in the two price stickiness frameworks if the steady state is efficient. Lombardo and Vestin (2008) have shown that, up to second order, Calvo pricing i) generates higher unconditional welfare losses than Rotemberg pricing even in an efficient steady state and ii) that for realistic model calibrations, conditional welfare losses of Calvo pricing are also higher in a distorted steady

[17]In the presence of trend inflation, differences arise already at first order (see e.g. Ascari and Rossi 2012). Analyzing the issues arising in this case is beyond the scope of the present paper.

state. In the following, we investigate whether these findings also hold for wage stickiness.

5.1 Setup

Our welfare analysis is based on the canonical model setup of Galí (2015, Chapter 6) described in the previous section, except that we now consider the four different versions of the labor market discussed above.

The relevant felicity function U_t is an aggregate over all agents j in the economy. In the case of EHL insurance scheme and Calvo price rigidities, we get

$$U_t = \int_0^1 U_t^j dj = Z_t \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{\left(\frac{N_t}{S_t^W}\right)^{1+\varphi}}{1+\varphi} X_t^W \right) , \tag{5.1}$$

where S_t^W and X_t^W are auxiliary variables related to wage dispersion whose recursive laws of motion are given in Appendix D. These terms reflect the fact that wage dispersion in the EHL framework results in cross-sectional differences in hours worked that are immediately welfare reducing.

For the SGU Calvo and the two Rotemberg setups, we obtain

$$U_t = \int_0^1 U_t^j dj = Z_t \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right) , \tag{5.2}$$

as the symmetric equilibrium of the Rotemberg and the fact that household members supply a homogeneous labor good to unions in the SGU Calvo case considerably simplify aggregation.

5.2 Theoretical results

For our theoretical results, we first make an assumption that allows us to compare welfare across the four different frameworks while assigning them the same first-order dynamics.

Assumption 1 (Identical first-order dynamics)**.** *Assume that the slope of the linearized Wage Phillips Curve is identical in the four setups.*

Our first result concerns welfare under Rotemberg wage setting in the EHL and SGU framework. As the two setups are isomorphic, they also result in the same welfare losses:

Proposition 1 (Welfare under Rotemberg pricing). *Under Assumption 1, the welfare losses from Rotemberg wage stickiness are identical in the EHL and SGU setup.*

Proof. The result immediately follows from the identical aggregate utility function (5.2) and the identical first order condition governing the wage setting dynamics (see equation (B.16)), which proves that the two setups are isomorphic. □

We now introduce an additional assumption that allows for clean analytical results and a better comparison to the findings from the price setting literature. We will relax it in the next section.

Assumption 2 (Efficient steady state). *Assume that the steady state is efficient, i.e. appropriate subsidies counteract the monopolistic distortion in the goods and labor markets and there is no trend inflation.*[18]

Our second result confirms that the finding of identical conditional welfare losses from inflation variability in the Calvo vs. Rotemberg price setting literature transfers to the welfare losses from wage inflation variability in the Calvo vs. Rotemberg wage setting:

Proposition 2 (Conditional welfare). *Under Assumptions 1 and 2, conditional on initial wage dispersion being zero in the Calvo wage setting framework, welfare losses from wage stickiness are identical up to second order in the Rotemberg and Calvo wage setting framework for both the EHL and SGU setup.*

Proof. See Appendix D. □

However, this proposition only refers to conditional welfare. As is well-known, the Calvo framework introduces an additional state variable in the form of wage dispersion. This additional state variable is a source of fundamental differences between the two frameworks. Wage dispersion in the stochastic equilibrium is on average different from

[18] We assume that firms are paid a subsidy to counteract the distortion in the product market, while households pay a tax on their wage to counteract the distortion in the labor market. Due to the slope of the Wage Phillips Curve in the Rotemberg case depending on labor taxes, it matters how the subsidy to counteract monopolistic distortions is introduced. In the Calvo framework, one could simply assume that firms are paid a subsidy that counteracts both the monopolistic distortion in the product and in the labor market (see e.g. Galí 2015, Chapter 6).

zero, with its unconditional mean increasing in the variance of wage inflation. Thus, from an unconditional perspective, the on-average non-zero wage dispersion in the Calvo framework causes welfare losses that are not present in the Rotemberg framework. The next proposition summarizes the differences in unconditional welfare:

Proposition 3 (Unconditional welfare)**.** *Even if Assumptions 1 and 2 are satisfied, the Calvo wage setting framework up to second order produces higher unconditional welfare losses than the Rotemberg framework.*

Proof. See Appendix D. □

This results is also not surprising as it again mirrors the findings from the price setting literature (see Lombardo and Vestin 2008). Our last proposition concerns the different unconditional welfare implications of the SGU and EHL frameworks. The central difference between the two is whether households supply an idiosyncratic labor service and thus whether the idiosyncratic or aggregate marginal rate of substitution is used to value the real wage. We already saw in section (3) that this distinction does not matter in the symmetric Rotemberg equilibrium, because all workers are alike and the idiosyncratic MRS coincides with the aggregate one. The situation is different with Calvo wage setting where wage dispersion also creates a heterogeneity in hours worked that is welfare-relevant. This can be seen from the presence of the wage dispersion terms S_t^w and X_t^w in the aggregate felicity function in equation (5.2). The negative aggregate utility effect from a dispersion in hours worked in the EHL setup comes on top of the welfare loss caused by the inefficiency in production introduced by wage dispersion, which appears in both the EHL and SGU frameworks. Thus, for a given amount of initial wage dispersion, the welfare losses in the SGU framework are bigger. However, under Assumption 1, the EHL framework produces a smaller unconditionally expected wage dispersion, because the unconditional mean of wage dispersion is a function of the Calvo parameter. As the slope of the Wage Phillips Curve in the EHL framework has an additional term $1 + \varepsilon_w \varepsilon_{tot}^{mrs}$ compared to the SGU framework, the required amount of Calvo wage stickiness to generate a particular slope of the Wage Phillips Curve is lower in the EHL setup than in the SGU setup. It turns out that for an efficient steady state the latter effect on the average wage dispersion always dominates the additional utility effect. Only if workers either do not dislike fluctuations in hours worked, i.e. if the Frisch elasticity of labor supply is infinite, or labor services are perfect complements ($\varepsilon_w = 0$) so that there are no differences in demand across varieties, are the welfare losses under SGU and EHL Calvo identical. In this case, the mean wage dispersion is the same, because the Calvo parameters coincide, and the dispersion in hours worked is only welfare relevant via its effect on aggregate output, which is identical in both setups.

Proposition 4 (Unconditional welfare: SGU vs. EHL)**.** *Under Assumptions 1 and 2, the unconditional welfare losses under Rotemberg wage setting are identical in the EHL and SGU framework, but the losses from Calvo wage setting in the SGU case are bigger than in the EHL case if* $\varepsilon_w \varepsilon_{tot}^{mrs} > 0$*. If* $\varepsilon_w \varepsilon_{tot}^{mrs} = 0$*, the welfare losses are identical.*

Proof. The first part regarding the Rotemberg wage setting immediately follows from Proposition 1. For the Calvo case, see Appendix D. □

After these qualitative results, we will next turn to a numerical evaluation of the welfare differences, which will also allow us to relax Assumption 2.

5.3 Numerical Evaluation

The exercise we conduct is in the spirit of Galí (2015, Chapter 6.5). We consider a number of simple monetary policy rules and quantitatively evaluate their impact on welfare, differentiating between Calvo and Rotemberg wage rigidities and EHL and SGU insurance schemes. In the experiments, fluctuations are either driven by the technology shock process A_t or the demand shock process Z_t, each with innovations with 1% standard deviation. The former shock is one that affects natural output, while the latter is a pure demand shock that can in principle be fully stabilized by monetary policy. The model calibration is shown in Table 3. We take the EHL Calvo case as the benchmark, resulting in a slope of the Wage Phillips Curve of 0.0037 that we keep fixed throughout.

Denote with V_0 expected lifetime utility in a particular specification:

$$V_0 = E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, N_t\right) \ . \tag{5.3}$$

We use as our welfare measure the fraction of flex-price consumption a household would be willing to give up in order to be indifferent to living under the alternative specification.[19] Thus, the permanent consumption loss λ is implicitly defined by

$$V_0 = E_0 \sum_{t=0}^{\infty} \beta^t U\left((1-\lambda) C_t^{nat}, N_t^{nat}\right) \ , \tag{5.4}$$

where the superscript indicates the natural level of the variables in the flex-price economy. In line with the discussion in the previous section, we consider both the conditional version λ_{cond} of this measure where expectations are taken conditional on being in a steady state with zero price and wage dispersion as well as the unconditional version λ_{unc}.

In our evaluation of welfare differences across the four different labor market specifications, we follow Galí (2015) and consider two different sets of monetary policy. The first set are "strict targeting rules" that require that either price inflation or wage inflation or a composite inflation measure to be zero at all times:

$$\Pi_t^k = 0, k \in \{p, w, c\} \ , \tag{5.5}$$

where the composite inflation measure is a weighted average

$$\Pi_t^c \equiv \left(\Pi_t^p\right)^{\vartheta} \left(\Pi_t^w\right)^{1-\vartheta} \tag{5.6}$$

and the weight $\vartheta \equiv \frac{\Lambda_p}{\Lambda_p + \Lambda_w}$ is given by the relative slopes of the linearized Price and Wage Phillips Curves, Λ_p and Λ_w, respectively.

The second set consists of Taylor rule type "flexible targeting rates" that take the form

$$R_t = R\left(\Pi_t^k\right)^{1.5}, k \in \{p, w, c\} \ , \tag{5.7}$$

where R denotes the steady-state nominal interest rate. The model is solved using

[19] Note that in the previous subsection, we used welfare losses relative to the steady-state allocation, while here we rely on differences relative to the flex-price allocation. However, as the steady state and the flex-price allocation are the same in all four setups, this distinction is inconsequential and involves only a renormalization.

second-order perturbation techniques in Dynare (Adjemian, Bastani, Juillard, Karamé, Maih, Mihoubi, Perendia, Pfeifer, Ratto, and Villemot 2011). Unconditional lifetime utility is computed as the theoretical mean based on the first-order terms of the second-order approximation to the nonlinear model,[20] resulting in a second-order accurate welfare measure (see e.g. Kim, Kim, Schaumburg, and Sims 2008). Conditional welfare is evaluated at the deterministic steady state.

Table 4 displays the results for the case of an efficient steady state. The upper left panel displays the results from the EHL Calvo setup, which corresponds exactly to the case considered in Galí (2015, Table 6.1).[21] The table displays both the unconditional and conditional (on the deterministic steady state) permanent consumption equivalent relative to the flex-price equilibrium.

Consistent with our theoretical results, all four setups produce the same conditional welfare losses when the steady state is efficient. We also see that the two Rotemberg setups produce the same unconditional welfare losses, which are smaller than the unconditional losses under Calvo. Among the Calvo setups, the EHL framework produces smaller unconditional losses than the SGU framework. This is driven by the fact that the SGU framework requires a Calvo wage adjustment parameter $\theta_w = 0.9458$ to generate the same Wage Phillips Curve slope as the EHL framework with $\theta_w = 0.75$. Despite these qualitative differences, the quantitative differences in unconditional welfare between the best (Rotemberg) and worst setup (Calvo SGU) are small, amounting to just 0.06% of consumption under the worst policy (strict price targeting under technology shocks)

Table 5 displays the results for the case of an inefficient steady state where monopolistic competition in goods and labor markets drives a wedge between the marginal rate of substitution and the marginal product of labor.

Three things are notable. First, in line with Proposition 1, the conditional and unconditional welfare losses from Rotemberg wage setting are identical in the EHL and

[20] The nonlinear first-order conditions can be produced from the replication files by making use of Dynare's LaTeX capabilities.

[21] Table 7 in Appendix D displays the variance of log price inflation, log wage inflation, and of the log output gap, because with an efficient steady state welfare up to second order can be expressed as a linear function of these three terms only.

Table 4: Welfare: Efficient Steady State

	EHL Calvo						EHL Rotemberg					
	Strict Targeting			Flexible Targeting			Strict Targeting			Flexible Targeting		
	Price	Wage	Comp.	Price	Wage	Comp.	Price	Wage	Comp.	Price	Wage	Comp.
	Technology Shock											
λ_{unc}	0.802	0.041	0.035	0.489	0.313	0.315	0.792	0.041	0.035	0.482	0.309	0.311
λ_{cond}	0.773	0.038	0.032	0.450	0.295	0.297	0.773	0.038	0.032	0.450	0.295	0.297
	Demand Shock											
λ_{unc}	0.000	0.000	0.000	0.062	0.069	0.067	0.000	0.000	0.000	0.062	0.068	0.067
λ_{cond}	0.000	0.000	0.000	0.061	0.067	0.066	0.000	0.000	0.000	0.061	0.067	0.066
	SGU Calvo						SGU Rotemberg					
	Technology Shock											
λ_{unc}	0.849	0.041	0.035	0.527	0.331	0.334	0.792	0.041	0.035	0.482	0.309	0.311
λ_{cond}	0.773	0.038	0.032	0.450	0.295	0.297	0.773	0.038	0.032	0.450	0.295	0.297
	Demand Shock											
λ_{unc}	0.000	0.000	0.000	0.064	0.071	0.070	0.000	0.000	0.000	0.062	0.068	0.067
λ_{cond}	0.000	0.000	0.000	0.061	0.067	0.066	0.000	0.000	0.000	0.061	0.067	0.066

SGU setup. Second, the differences between Calvo and Rotemberg wage setting are now more pronounced, reaching a consumption equivalent difference of 0.1% between EHL Calvo and Rotemberg for strict price targeting with technology shocks. Third, the welfare ranking between EHL Calvo and SGU Calvo reverses compared to the efficient steady state, with SGU Calvo producing smaller welfare losses than EHL. That difference is particularly pronounced conditional on being in the deterministic steady state. Here, welfare under SGU Calvo is very close to the one in the Rotemberg setup, while EHL Calvo has losses that are higher by up to 0.08% of flex-price consumption.

6 Conclusion

We have provided applied researchers with guidance on how to translate a Calvo wage duration into an implied Rotemberg wage adjustment cost parameter by using the equivalence of their setups at first order. In doing so, we have shown that both the presence of labor taxation and the assumed household insurance scheme matter greatly for this mapping, giving rise to differences of up to one order of magnitude. Our results account for the inclusion of wage indexing, habit formation in consumption, and the presence of fixed costs in production, features commonly used in medium-scale New Keynesian DSGE models.

In the second part of the paper we investigated the second-order implications of Rotemberg vs. Calvo wage setting by turning to a welfare evaluation. Despite first-order equivalence, Calvo and Rotemberg wage setting generally produce different welfare implications, which are a second-order property. We showed that both wage setting schemes are equivalent conditional on initial wage dispersion being zero and the steady state being efficient. If these assumptions are not satisfied, Calvo wage setting generally produces higher welfare losses.

The present work restricted itself to the case where the first-order dynamics of the different wage setting schemes are identical. An interesting avenue for future research would be to investigate what happens in the presence of trend inflation. Due to non-zero

Table 5: Welfare: Inefficient Steady State

	EHL Calvo						EHL Rotemberg					
	Strict Targeting			Flexible Targeting			Strict Targeting			Flexible Targeting		
	Price	Wage	Comp.	Price	Wage	Comp.	Price	Wage	Comp.	Price	Wage	Comp.
	Technology Shock											
λ_{unc}	0.794	0.041	0.035	0.482	0.309	0.311	0.690	0.041	0.034	0.392	0.265	0.265
λ_{cond}	0.767	0.038	0.032	0.454	0.296	0.297	0.672	0.038	0.032	0.373	0.255	0.255
	Demand Shock											
λ_{unc}	0.000	0.000	0.000	0.062	0.068	0.067	0.000	0.000	0.000	0.057	0.061	0.061
λ_{cond}	0.000	0.000	0.000	0.061	0.067	0.066	0.000	0.000	0.000	0.057	0.061	0.060
	SGU Calvo						SGU Rotemberg					
	Technology Shock											
λ_{unc}	0.744	0.041	0.034	0.441	0.289	0.291	0.690	0.041	0.034	0.392	0.265	0.265
λ_{cond}	0.687	0.038	0.032	0.389	0.263	0.264	0.672	0.038	0.032	0.373	0.255	0.255
	Demand Shock											
λ_{unc}	0.000	0.000	0.000	0.060	0.065	0.064	0.000	0.000	0.000	0.057	0.061	0.061
λ_{cond}	0.000	0.000	0.000	0.057	0.062	0.061	0.000	0.000	0.000	0.057	0.061	0.060

steady-state Calvo wage dispersion and Rotemberg resource costs, already the first-order dynamics differ. Based on the work of Ascari and Rossi (2012) for price setting, it stands to be expected that the different wage setting schemes give rise to significant differences in, e.g., welfare and determinacy properties.

References

Adjemian, Stéphane et al. (2011). "Dynare: reference manual, version 4". Dynare Working Papers 1. CEPREMAP.

Ascari, Guido and Lorenza Rossi (2012). "Trend inflation and firms price-setting: Rotemberg versus Calvo". *Economic Journal 122* (563), 1115–1141.

Ascari, Guido and Argia M. Sbordone (2014). "The macroeconomics of trend inflation". *Journal of Economic Literature 52* (3), 679–739.

Backus, David K., Patrick J. Kehoe, and Finn E. Kydland (1992). "International real business cycles". *Journal of Political Economy 100* (4), 745–75.

Barattieri, Alessandro, Susanto Basu, and Peter Gottschalk (2014). "Some evidence on the importance of sticky wages". *American Economic Journal: Macroeconomics 6* (1), 70–101.

Basu, Susanto and Brent Bundick (2017). "Uncertainty shocks in a model of effective demand". *Econometrica* (85) (3), 937–958.

Bils, Mark and Peter J. Klenow (2004). "Some evidence on the importance of sticky prices". *Journal of Political Economy 112* (5), 947–985.

Born, Benjamin, Alexandra Peter, and Johannes Pfeifer (2013). "Fiscal news and macroeconomic volatility". *Journal of Economic Dynamics and Control 37* (12), 2582–2601.

Born, Benjamin and Johannes Pfeifer (2014). "Policy risk and the business cycle". *Journal of Monetary Economics 68*, 68–85.

——— (2017). "Uncertainty-driven business cycles: assessing the markup channel". CEPR Discussion Papers 11745.

Calvo, Guillermo A. (1983). "Staggered prices in a utility-maximizing framework". *Journal of Monetary Economics 12* (3), 383–398.

Chetty, Raj, Adam Guren, Day Manoli, and Andrea Weber (2011). "Are micro and macro labor supply elasticities consistent? A review of evidence on the intensive and extensive margins". *American Economic Review 101* (3), 471–75.

Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans (2005). "Nominal rigidities and the dynamic effects of a shock to monetary policy". *Journal of Political Economy 113* (1), 1–45.

Christiano, Lawrence J., Roberto Motto, and Massimo Rostagno (2014). "Risk shocks". *American Economic Review 104* (1), 37–65.

Chugh, Sanjay K. (2006). "Optimal fiscal and monetary policy with sticky wages and sticky prices". *Review of Economic Dynamics* (9), 683–714.

Damjanovic, Tatiana and Charles Nolan (2011). "Second-order approximation to the Rotemberg model around a distorted steady state". *Economics Letters 110* (2), 132–135.

Dixit, Avinash K. and Joseph E. Stiglitz (1977). "Monopolistic competition and optimum product diversity". *American Economic Review 67* (3), 297–308.

Dmitriev, Mikhail and Jonathan Hoddenbagh (2017). "The financial accelerator and the optimal state-dependent contract". *Review of Economic Dynamics 24*, 43–65.

Erceg, Christopher J., Dale W. Henderson, and Andrew T. Levin (2000). "Optimal monetary policy with staggered wage and price contracts". *Journal of Monetary Economics 46* (2), 281–313.

Faia, Ester, Wolfgang Lechthaler, and Christian Merkl (2014). "Labor selection, turnover costs, and optimal monetary policy". *Journal of Money, Credit and Banking 46* (1), 115–144.

Fernández-Villaverde, Jesús, Pablo A. Guerrón-Quintana, Keith Kuester, and Juan F. Rubio-Ramírez (2015). "Fiscal volatility shocks and economic activity". *Amercian Economic Review 105* (11), 3352–3384.

Fernández-Villaverde, Jesús, Pablo A. Guerrón-Quintana, Juan F. Rubio-Ramírez, and Martín Uribe (2011). "Risk matters: the real effects of volatility shocks". *American Economic Review 101* (6), 2530–61.

Galí, Jordi (2011). "The return of the wage Phillips curve". *Journal of the European Economic Association 9* (3), 436–461.

——— (2015). *Monetary policy, inflation and the business cycle*. 2nd ed. Princeton University Press.

Greenwood, Jeremy, Zvi Hercowitz, and Gregory W. Huffman (1988). "Investment, capacity utilization, and the real business cycle". *American Economic Review 78* (3), 402–17.

Hagedorn, Marcus, Iourii Manovskii, and Kurt Mitman (2018). "The fiscal multiplier". Mimeo.

Heer, Burkhard, Torben Klarl, and Alfred Maussner (2012). "Asset pricing implications of a New Keynesian model: a note". CESifo Working Paper Series 4041.

Johannsen, Benjamin K. (2014). "When are the effects of fiscal policy uncertainty large?" Finance and Economics Discussion Series 2014-40. Federal Reserve Board.

Jones, John Bailey (2002). "Has fiscal policy helped stabilize the postwar U.S. economy?" *Journal of Monetary Economics 49* (4), 709–746.

Keen, Benjamin and Yongsheng Wang (2007). "What is a realistic value for price adjustment costs in New Keynesian models?" *Applied Economics Letters 14* (11), 789–793.

Kim, Jinill, Sunghyun Kim, Ernst Schaumburg, and Christopher A. Sims (2008). "Calculating and using second order accurate solutions of discrete time dynamic equilibrium models". *Journal of Economic Dynamics and Control 32* (11), 3397–3414.

King, Robert G. and Sergio T. Rebelo (1999). "Resuscitating real business cycles". *Handbook of Macroeconomics*. Ed. by John B. Taylor and Michael Woodford. Vol. 1, Part A. Elsevier, 927–1007.

Le Bihan, Hervé, Jérémi Montornès, and Thomas Heckel (2012). "Sticky wages: evidence from quarterly microeconomic data". *American Economic Journal: Macroeconomics 4* (3), 1–32.

Lombardo, Giovanni and David Vestin (2008). "Welfare implications of Calvo vs. Rotemberg-pricing assumptions". *Economics Letters 100* (2), 275–279.

Mumtaz, Haroon and Francesco Zanetti (2013). "The impact of the volatility of monetary policy shocks". *Journal of Money, Credit and Banking 45* (4), 535–558.

Nakamura, Emi and Jón Steinsson (2008). "Five facts about prices: a reevaluation of menu cost models". *Quarterly Journal of Economics 123* (4), 1415–1464.

Nath, Rahul (2015). "Asset pricing in a New Keynesian model with sticky wages". Mimeo. Oxford University.

Nisticó, Salvatore (2007). "The welfare loss from unstable inflation". *Economics Letters 96* (1), 51–57.

Pariés, Matthieu Darracq, Christoffer Kok Sørensen, and Diego Rodriguez-Palenzuela (2011). "Macroeconomic propagation under different regulatory regimes: evidence from an estimated DSGE model for the euro area". *International Journal of Central Banking 7* (4), 49–113.

Plante, Michael, Alexander W. Richter, and Nathaniel A. Throckmorton (forthcoming). "The zero lower bound and endogenous uncertainty". *Economic Journal*, n/a–n/a.

Richter, Alexander W. and Nathaniel A. Throckmorton (2016). "Is Rotemberg pricing justified by macro data?" *Economics Letters 149*, 44–48.

Roberts, John M. (1995). "New Keynesian economics and the Phillips Curve". *Journal of Money, Credit and Banking 27* (4), 975–84.

Rotemberg, Julio J. (1982). "Sticky Prices in the United States". *Journal of Political Economy 90* (6), 1187–1211.

Sbordone, Argia M. (2006). "U.S. wage and price dynamics: a limited-information approach". *International Journal of Central Banking 2* (3), 155–191.

Schmitt-Grohé, Stephanie and Martín Uribe (2006a). "Comparing two variants of Calvo-type wage stickiness". NBER Working Paper 12740.

——— (2006b). "Optimal fiscal and monetary policy in a medium-scale macroeconomic model". *NBER Macroeconomics Annual 2005, Volume 20*, 383–462.

Smets, Frank and Rafael Wouters (2007). "Shocks and frictions in US business cycles: a Bayesian DSGE approach". *American Economic Review 97* (3), 586–606.

Taylor, John B. (1999). "Staggered price and wage setting in macroeconomics". *Handbook of Macroeconomics*. Ed. by John B. Taylor and Michael Woodford. Vol. 1, Part B. Elsevier, 1009–1050.

Trigari, Antonella (2009). "Equilibrium unemployment, job flows, and inflation dynamics". *Journal of Money, Credit and Banking 49* (1), 1–32.

Uhlig, Harald (2007). “Explaining asset prices with external habits and wage rigidities in a DSGE model”. *American Economic Review 97* (2), 239–243.

Woodford, Michael (1999). “Revolution and evolution in twentieth-century macroeconomics”. Mimeo. Columbia University.

Yun, Tack (1996). “Nominal price rigidity, money supply endogeneity, and business cycles”. *Journal of Monetary Economics 37* (2), 345–370.

A EHL algebra

A.1 Calvo

The Lagrangian for the EHL Calvo setup is given by

$$L = \sum_{k=0}^{\infty} (\beta\theta_w)^k\, E_t \Bigg[U\left(C_{t+k|t}\ , \left(\frac{\Gamma^{ind}_{t,t+k} W^*_t}{W_{t+k}} \right)^{-\varepsilon_w} N^d_{t+k}, \cdot \right) \\ - \lambda_{t+k|t} \left\{ (1+\tau^c_{t+k}) P_{t+k} C_{t+k|t} - (1-\tau^n_{t+k}) \Gamma^{ind}_{t,t+k} W^*_t \left(\frac{\Gamma^{ind}_{t,t+k} W^*_t}{W_{t+k}} \right)^{-\varepsilon_w} N^d_{t+k} - X_{t+k} \right\} \Bigg] , \tag{A.1}$$

where $\lambda_{t+k|t}$ is the Lagrange multiplier and the j index has been suppressed. The FOC for consumption is given by

$$(1+\tau^c_{t+k})\lambda_{t+k|t} P_{t+k} = V_{C,t+k|t}\ . \tag{A.2}$$

The FOC for W^*_t is given by

$$0 = \sum_{k=0}^{\infty} (\beta\theta_w)^k\, E_t \Bigg[U_N \left(C_{t+k|t}, \left(\frac{\Gamma^{ind}_{t,t+k} W^*_t}{W_{t+k}} \right)^{-\varepsilon_w} N^d_{t+k}, \cdot \right) (-\varepsilon_w) \left(\frac{\Gamma^{ind}_{t,t+k} W^*_t}{W_{t+k}} \right)^{-\varepsilon_w} \frac{N^d_{t+k}}{W^*_t} \\ + \lambda_{t+k|t} \left\{ (1-\varepsilon_w)(1-\tau^n_{t+k}) \Gamma^{ind}_{t,t+k} \left(\frac{\Gamma^{ind}_{t,t+k} W^*_t}{W_{t+k}} \right)^{-\varepsilon_w} N^d_{t+k} \right\} \Bigg] , \tag{A.3}$$

where U_N denotes the partial derivative of the felicity function with respect to N. Using

$$N^j_{t+k|t} = \left(\frac{W^j_{t+k|t}}{W_{t+k}} \right)^{-\varepsilon_w} N^d_{t+k}\ , \tag{2.2}$$

$$W^j_{t+k|t} = \Gamma^{ind}_{t,t+k} W^*_t\ , \tag{2.3}$$

and suppressing the arguments of the felicity function this can be rewritten as:

$$0 = \sum_{k=0}^{\infty} (\beta\theta_w)^k\, E_t \left[N_{t+k|t} \lambda_{t+k|t} \left(\frac{\varepsilon_w}{\varepsilon_w - 1} \frac{U_{N,t+k|t}}{\lambda_{t+k|t}} + (1-\tau^n_{t+k}) \Gamma^{ind}_{t,t+k} W^*_t \right) \right] . \tag{A.4}$$

Replacing $\lambda_{t+k|t}$ using (A.2) yields

$$0 = \sum_{k=0}^{\infty} (\beta\theta_w)^k\, E_t \left[N_{t+k|t} \frac{V_{C,t+k|t}(1-\tau^n_{t+k})}{(1+\tau^c_{t+k})} \left(\frac{\varepsilon_w}{\varepsilon_w - 1} \frac{U_{N,t+k|t}(1+\tau^c_{t+k})}{V_{C,t+k|t}(1-\tau^n_{t+k})} + \frac{\Gamma^{ind}_{t,t+k} W^*_t}{P_{t+k}} \right) \right] . \tag{A.5}$$

Making use of the definition of the after-tax marginal rate of substitution

$$MRS_{t+k|t} = - \frac{\left(1+\tau^c_{t+k}\right)}{\left(1-\tau^n_{t+k}\right)} \frac{U_{N,t+k|t}}{V_{C,t+k|t}} \tag{2.5}$$

this yields

$$0 = \sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \left[N_{t+k|t} \frac{V_{C,t+k|t}(1-\tau^n_{t+k})}{(1+\tau^c_{t+k})} \left(\frac{\varepsilon_w}{\varepsilon_w - 1} MRS_{t+k|t} - \frac{\Gamma^{ind}_{t,t+k} W^*_t}{P_{t+k}} \right) \right] . \tag{A.6}$$

Performing a log-linearization around the deterministic steady state yields[22]

$$0 = \sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \left[\frac{\varepsilon_w}{\varepsilon_w - 1} MRS \times \widehat{MRS}_{t+k|t} - \Gamma^{ind}_k \frac{W^*}{P} \left(\hat{W}^*_t - \hat{P}_{t+k} + \hat{\Gamma}^{ind}_{t,t+k} \right) \right] \tag{A.7}$$

or

$$\hat{W}^*_t = (1-\beta\theta_w) \sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \left[\widehat{MRS}_{t+k|t} + \hat{P}_{t+k} - \hat{\Gamma}^{ind}_{t,t+k} \right] . \tag{2.6}$$

Expand $MRS_{t+k|t}(C_{t+k|t}, N_{t+k|t})$ by the average MRS in the economy

$$MRS_{t+k|t} = \frac{MRS_{t+k|t}}{MRS_{t+k}} MRS_{t+k} \tag{A.8}$$

and log-linearize around the deterministic steady state:[23]

$$\widehat{MRS}_{t+k|t} = \varepsilon^{mrs}_c \left(\hat{C}_{t+k|t} - \hat{C}_{t+k} \right) + \varepsilon^{mrs}_n \left(\hat{N}_{t+k|t} - \hat{N}_{t+k} \right) + \widehat{MRS}_{t+k} , \tag{A.9}$$

where $\varepsilon^{mrs}_c \equiv (MRS_C \times C)/MRS$ and $\varepsilon^{mrs}_n \equiv (MRS_N \times N)/MRS$ denote the elasticities of the MRS with respect to C and N, respectively. Due to the required assumption of complete markets and equal initial wealth, marginal utilities are equal across households. Therefore

$$V_{C,t+k} = V_{C,t+k|t} \tag{A.10}$$

and log-linearized

$$V_{CC} C \hat{C}_{t+k} + V_{CN} N \hat{N}_{t+k} = V_{CC} C \hat{C}_{t+k|t} + V_{CN} N \hat{N}_{t+k|t} . \tag{A.11}$$

Rearranging

$$V_{CC} C \left(\hat{C}_{t+k|t} - \hat{C}_{t+k} \right) = -V_{CN} N \left(\hat{N}_{t+k|t} - \hat{N}_{t+k} \right) \tag{A.12}$$

and plugging into (A.9) yields

$$\widehat{MRS}_{t+k|t} = \widehat{MRS}_{t+k} + \underbrace{\left[-\frac{V_{CN} N}{V_{CC} C} \varepsilon^{mrs}_c + \varepsilon^{mrs}_n \right]}_{\equiv \varepsilon^{mrs}_{tot}} \left(\hat{N}_{t+k|t} - \hat{N}_{t+k} \right) . \tag{2.7}$$

[22] Depending on the exact conduct of monetary policy, e.g., in case of an interest rate rule, the steady state of nominal variables like P_t and W_t may not be well-defined (see e.g. Galí 2015). Linearization in this case can be interpreted as being done around the long-run trend of the nominal variables. Linearization around a proper steady state would involve rewriting the problem in terms of stationary variables like the real wage W_t/P_t and inflation rates, but would yield the same results as trend changes only appear as ratios and therefore cancel out.

[23] The computational steps here follow Sbordone (2006). If the MRS depends on additional variables like housing or durables, the same approach can be followed to replace the idiosyncratic MRS by the aggregate one.

This together with the linearized labor demand

$$\hat{N}_{t+k|t} = -\varepsilon_w \left(\hat{\Gamma}^{ind}_{t,t+k} + \hat{W}^*_t - \hat{W}_{t+k} \right) + \hat{N}^d_{t+k} \tag{A.13}$$

and the fact that up to first-order wage dispersion is zero and therefore $N^d_{t+k} = N_{t+k}$ can be used to express the idiosyncratic MRS as

$$\widehat{MRS}_{t+k|t} = \widehat{MRS}_{t+k} - \varepsilon_w \varepsilon^{mrs}_{tot} \left(\hat{\Gamma}^{ind}_{t,t+k} + \hat{W}^*_t - \hat{W}_{t+k} \right) . \tag{A.14}$$

Plug into (2.6) to get

$$\hat{W}^*_t = (1 - \beta\theta_w) \left(\hat{W}_t + \frac{1}{1 + \varepsilon_w \varepsilon^{mrs}_{tot}} \left(\widehat{MRS}_t - \left(\hat{W}_t - \hat{P}_t \right) \right) \right) + \beta\theta_w E_t \left(\hat{W}^*_{t+1} - \hat{\Gamma}^{ind}_{t,t+1} \right) . \tag{A.15}$$

where we have made use of $\hat{\Gamma}^{ind}_{t,t+k} = \hat{\Gamma}^{ind}_{t,t+1} + \hat{\Gamma}^{ind}_{t+1,t+k}$ and $\hat{\Gamma}^{ind}_{t,t} = 0$.

Next, plug in from the linearized LOM for wages in the economy

$$\hat{W}^*_t = \frac{1}{1 - \theta_w} \hat{W}_t - \frac{\theta_w}{1 - \theta_w} (\hat{\Gamma}^{ind}_{t-1,t} + \hat{W}_{t-1}) \tag{A.16}$$

to get

$$\begin{aligned} \frac{1}{1-\theta_w}\hat{W}_t - \frac{\theta_w}{1-\theta_w}\left(\hat{\Gamma}^{ind}_{t-1,t} + \hat{W}_{t-1}\right) \;&=\; (1-\beta\theta_w)\left(\hat{W}_t - \frac{1}{1+\varepsilon_w\varepsilon^{mrs}_{tot}}\hat{\mu}^w_t\right) \\ &+ \beta\theta_w E_t\left(-\hat{\Gamma}^{ind}_{t,t+1} + \frac{1}{1-\theta_w}\hat{W}_{t+1} - \frac{\theta_w}{1-\theta_w}\left(\hat{\Gamma}^{ind}_{t,t+1} + \hat{W}_t\right)\right) . \end{aligned} \tag{A.17}$$

Now add 0 to the left-hand side and expand the right-hand side:

$$\begin{aligned} &\frac{1}{1-\theta_w}\hat{W}_t - \frac{\theta_w}{1-\theta_w}\left(\hat{\Gamma}^{ind}_{t-1,t} + \hat{W}_{t-1}\right) + \left(\frac{1}{1-\theta_w}\hat{W}_{t-1} - \frac{1}{1-\theta_w}\hat{W}_{t-1}\right) \\ &= (1-\beta\theta_w)\hat{W}_t - \beta\theta_w\left(\frac{\theta_w}{1-\theta_w}\left(E_t\hat{\Gamma}^{ind}_{t,t+1} + \hat{W}_t\right) + \frac{1-\theta_w}{1-\theta_w}E_t\hat{\Gamma}^{ind}_{t,t+1}\right) \\ &+ \frac{\beta\theta_w}{1-\theta_w}E_t\left(\hat{W}_{t+1}\right) - \frac{(1-\beta\theta_w)}{1+\varepsilon_w\varepsilon^{mrs}_{tot}}\hat{\mu}^w_t . \end{aligned} \tag{A.18}$$

Factor the left-hand side and collect terms related to W_t on the right-hand side

$$\begin{aligned} &\frac{1}{1-\theta_w}\left(\hat{W}_t - \hat{W}_{t-1}\right) + \hat{W}_{t-1} - \frac{\theta_w}{1-\theta_w}\hat{\Gamma}^{ind}_{t-1,t} \\ &= \underbrace{\left(\frac{1-\beta\theta_w - \theta_w(1-\beta\theta_w) - \beta\theta_w\theta_w}{1-\theta_w}\right)}_{1-\frac{\beta\theta_w}{1-\theta_w}}\hat{W}_t \\ &- \frac{\beta\theta_w}{1-\theta_w}E_t\hat{\Gamma}^{ind}_{t,t+1} + \frac{\beta\theta_w}{1-\theta_w}E_t\left(\hat{W}_{t+1}\right) - \frac{(1-\beta\theta_w)}{1+\varepsilon_w\varepsilon^{mrs}_{tot}}\hat{\mu}^w_t . \end{aligned} \tag{A.19}$$

Subtract W_t from both sides

$$\begin{aligned}
&\frac{1}{1-\theta_w}\left(\hat{W}_t - \hat{W}_{t-1}\right) - \frac{\theta_w}{1-\theta_w}\hat{\Gamma}^{ind}_{t-1,t} - \left(\hat{W}_t - \hat{W}_{t-1}\right) \\
&= \frac{\beta\theta_w}{1-\theta_w}E_t\left(\hat{W}_{t+1} - \hat{W}_t\right) - \frac{(1-\beta\theta_w)}{1+\varepsilon_w\varepsilon^{mrs}_{tot}}\hat{\mu}^w_t - \frac{\beta\theta_w}{1-\theta_w}E_t\hat{\Gamma}^{ind}_{t,t+1} \ .
\end{aligned} \tag{A.20}$$

Collecting terms:

$$\begin{aligned}
&\frac{\theta_w}{1-\theta_w}\left(\hat{W}_t - \hat{W}_{t-1}\right) - \frac{\theta_w}{1-\theta_w}\hat{\Gamma}^{ind}_{t-1,t} \\
&\quad = \frac{\beta\theta_w}{1-\theta_w}E_t\left(\hat{W}_{t+1} - \hat{W}_t\right) - \frac{(1-\beta\theta_w)}{1+\varepsilon_w\varepsilon^{mrs}_{tot}}\hat{\mu}^w_t - \frac{\beta\theta_w}{1-\theta_w}E_t\hat{\Gamma}^{ind}_{t,t+1} \ .
\end{aligned} \tag{A.21}$$

Solve for wage inflation:

$$\hat{\Pi}^w_t = \beta E_t\hat{\Pi}_{w,t+1} - \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w\left(1+\varepsilon_w\varepsilon^{mrs}_{tot}\right)}\hat{\mu}^w_t - \frac{\beta\theta_w}{1-\theta_w}E_t\hat{\Gamma}^{ind}_{t,t+1} + \frac{\theta_w}{1-\theta_w}\hat{\Gamma}^{ind}_{t-1,t} \ . \tag{2.8}$$

A.2 Rotemberg

The Lagrangian for the EHL Rotemberg setup is given by

$$L = \sum_{k=0}^{\infty}\beta^k E_t\left[\begin{array}{l}
U\left(C_{t+k}, \left(\frac{W^j_{t+k}}{W_{t+k}}\right)^{-\varepsilon_w} N^d_{t+k}\right) \\
-\lambda_{t+k}\left\{\begin{array}{l}
\left(1+\tau^c_{t+k}\right)P_{t+k}C^j_{t+k} - \left(1-\tau^n_{t+k}\right)W^j_{t+k}\left(\frac{W^j_{t+k}}{W_{t+k}}\right)^{-\varepsilon_w} N^d_{t+k} \\
+\frac{\phi_w}{2}\left(\frac{1}{\Gamma^{ind}_{t+k-1,t+k}}\frac{W^j_{t+k}}{W^j_{t+k-1}} - 1\right)^2 \Xi_{t+k} - X_{t+k}
\end{array}\right\}
\end{array}\right] . \tag{A.22}$$

The FOC for consumption is given by

$$(1+\tau^c_{t+k})\lambda^j_{t+k}P_{t+k} = V_{C,t+k} \ . \tag{A.23}$$

The corresponding FOC for the optimal wage is given by

$$\begin{aligned}
0 \ &= U_N\left(C^j_t, \left(\frac{W^j_t}{W_t}\right)^{-\varepsilon_w} N^d_t, \cdot\right)(-\varepsilon_w)\left(\frac{W^j_t}{W_t}\right)^{-\varepsilon_w}\frac{N^d_t}{W^j_t} \\
&+ \lambda^j_t\left\{(1-\varepsilon_w)(1-\tau^n_t)\left(\frac{W^j_t}{W_t}\right)^{-\varepsilon_w} N^d_t - \phi_w\left(\frac{1}{\Gamma^{ind}_{t-1,t}}\frac{W^j_t}{W^j_{t-1}} - 1\right)\frac{\Xi_t}{\Gamma^{ind}_{t-1,t}W^j_{t-1}}\right\} \\
&- E_t\lambda^j_{t+1}\left\{\phi_w\left(\frac{1}{\Gamma^{ind}_{t,t+1}}\frac{W^j_{t+1}}{W^j_t} - 1\right)(-1)\frac{W^j_{t+1}}{\left(W^j_t\right)^2}\frac{1}{\Gamma^{ind}_{t,t+1}}\Xi_{t+1}\right\} \ .
\end{aligned} \tag{A.24}$$

As there is no wage dispersion in the Rotemberg case, imposing symmetry means that $N_t^j = N_t^d = N_t$. Additionally substituting for λ_t from (A.23) and dividing by $V_{C,t}/(1+\tau_t^c)$, the above equation can be written as

$$\begin{aligned} 0 \ =\ & \frac{U_{N,t}}{V_{C,t}}(1+\tau_t^c)\left(-\varepsilon_w\right)\frac{N_t}{W_t} + \frac{1}{P_t}\left\{\left(1-\varepsilon_w\right)\left(1-\tau_t^n\right)N_t - \phi_w\left(\frac{1}{\Gamma_{t-1,t}^{ind}}\frac{W_t}{W_{t-1}} - 1\right)\frac{\Xi_t}{\Gamma_{t-1,t}^{ind}W_{t-1}}\right\} \\ & + E_t\beta\frac{V_{C,t+1}}{V_{C,t}}\frac{(1+\tau_t^c)}{(1+\tau_{t+1}^c)}\frac{1}{W_t}\left\{\phi_w\left(\frac{1}{\Gamma_{t,t+1}^{ind}}\frac{W_{t+1}}{W_t} - 1\right)\frac{W_{t+1}}{W_t}\frac{1}{\Gamma_{t,t+1}^{ind}}\frac{\Xi_{t+1}}{P_{t+1}}\right\} \ , \end{aligned} \tag{A.25}$$

or, dividing by N_t, multiplying by P_t, and making use of the definition of the after-tax MRS (2.5), as

$$\begin{aligned} 0 \ =\ & \varepsilon_w\frac{MRS_t}{\frac{W_t}{P_t}}(1-\tau_t^n) + \left\{\left(1-\varepsilon_w\right)\left(1-\tau_t^n\right) - \phi_w\left(\frac{\Pi_{w,t}}{\Gamma_{t-1,t}^{ind}} - 1\right)\Pi_t\frac{1}{N_t}\frac{1}{\Gamma_{t-1,t}^{ind}}\frac{\frac{\Xi_t}{P_t}}{\frac{W_{t-1}}{P_{t-1}}}\right\} \\ & + E_t\beta\frac{V_{C,t+1}}{V_{C,t}}\frac{(1+\tau_t^c)}{(1+\tau_{t+1}^c)}\frac{1}{N_t}\frac{1}{\frac{W_t}{P_t}}\left\{\phi_w\left(\frac{\Pi_{w,t+1}}{\Gamma_{t,t+1}^{ind}} - 1\right)\frac{\Pi_{w,t+1}}{\Gamma_{t,t+1}^{ind}}\frac{\Xi_{t+1}}{P_{t+1}}\right\} \ . \end{aligned} \tag{2.13}$$

Linearizing (2.13) around the steady state and making use of

$$\hat{\mu}_t^w \equiv \left(\hat{W}_t - \hat{P}_t\right) - \widehat{MRS}_t \tag{2.9}$$

and $\Gamma_1^{ind} = \Pi$ yields

$$\begin{aligned} 0 \ =\ & \varepsilon_w \underbrace{\frac{MRS}{\frac{W}{P}}}_{\frac{\varepsilon_w - 1}{\varepsilon_w}}(1-\tau_t^n)(-1)\hat{\mu}_t^w \\ & + \underbrace{\left[\varepsilon_w\frac{MRS}{\frac{W}{P}}(-\tau^n) + (1-\varepsilon_w)(-\tau^n)\right]}_{0}\hat{\tau}_t^n \\ & - \phi_w\underbrace{\left(\Pi^{-1}\Pi_w - 1\right)}_{0}\Pi\frac{1}{N}\Pi^{-1}\frac{\Xi^{real}}{W^{real}}\left(\hat{\Pi}_t - \hat{N}_t - \hat{\Gamma}_{t-1,t}^{ind} + \hat{\Xi}_t^{real} - \hat{W}_{t-1}^{real}\right) \\ & - \phi_w\Pi\frac{1}{N}\Pi^{-1}\frac{\Xi^{real}}{W^{real}}\Pi^{-1}\Pi_w\hat{\Pi}_{w,t} \\ & + E_t\beta\frac{1}{N}\frac{1}{W^{real}}\phi_w\underbrace{\left(\Pi^{-1}\Pi_w - 1\right)}_{0}\Pi^{-1}\Pi_w\Xi^{real}\left(\hat{V}_{C,t+1} - \hat{V}_{C,t} + \hat{\tau}_t^c - \hat{\tau}_{t+1}^c - \hat{N}_t - \hat{W}_t^{real} - \hat{\Xi}_{t+1}^{real}\right) \\ & + E_t\beta\frac{1}{N}\frac{1}{W^{real}}\phi_w\Pi^{-1}\Xi^{real}\left(2\Pi^{-1}\Pi_w^2 - \Pi_w\right)\hat{\Pi}_{w,t+1} \\ & + E_t\beta\frac{1}{N}\frac{1}{W^{real}}\phi_w\Pi_w\Xi^{real}\left(-2\Pi^{-2}\Pi_w + \Pi^{-1}\right)\hat{\Gamma}_{t,t+1}^{ind} \ . \end{aligned} \tag{A.26}$$

Simplifying and using the steady state relation $\Pi = \Pi_w$ yields

$$0 = (-1)\varepsilon_w \frac{\varepsilon_w - 1}{\varepsilon_w}(1-\tau^n)\hat{\mu}_t^w - \phi_w \underbrace{\frac{\Xi^{real}}{NW^{real}}}_{\frac{1}{\aleph}} \hat{\Pi}_{w,t} + E_t \beta \frac{\Xi^{real}}{NW^{real}} \phi_w \left(\hat{\Pi}_{w,t+1} - \hat{\Gamma}_{t,t+1}^{ind}\right) \tag{A.27}$$

and thus

$$\hat{\Pi}_{w,t} = \beta E_t \left(\hat{\Pi}_{w,t+1} - \hat{\Gamma}_{t,t+1}^{ind}\right) - \frac{(\varepsilon_w - 1)(1-\tau^n)\aleph}{\phi_w} \hat{\mu}_t^w \ . \tag{2.14}$$

B SGU algebra

B.1 Calvo

The associated Lagrangian is given by

$$\begin{aligned} L = \sum_{k=0}^{\infty} \beta^k E_t \Bigg[& U\left(C_{t+k}\ , N_{t+k}, \cdot\right) \\ - \lambda_{t+k} & \left\{ (1+\tau_{t+k}^c) P_{t+k} C_{t+k} - \left(1 - \tau_{t+k}^n\right) W_{t+k}^{\varepsilon_w} N_{t+k}^d \theta_w^k \left(\Gamma_{t,t+k}^{ind} W_t^*\right)^{1-\varepsilon_w} - X_{t+k} \right\} \Bigg], \end{aligned} \tag{B.1}$$

where in the budget constraint we have made use of

$$\begin{aligned} \int_0^1 W_{t+k}^j N_{t+k}^j dj &= \int_0^1 W_{t+k}^j \left(\frac{W_{t+k}^j}{W_{t+k}}\right)^{-\varepsilon_w} N_{t+k}^d dj \\ &= W_{t+k}^{\varepsilon_w} N_{t+k}^d \int_0^1 \left(W_{t+k}^j\right)^{1-\varepsilon_w} dj = W_{t+k}^{\varepsilon_w} N_{t+k}^d \left(\theta_w^k \left(\Gamma_{t,t+k}^{ind} W_t^*\right)^{1-\varepsilon_w} + \left(1-\theta_w^k\right) X_{1,t+k}\right) . \end{aligned} \tag{B.2}$$

The last term, $X_{1,t+k}$, captures the wage level in the other labor markets where price resetting has taken place. Hence, it is independent of W_t^* and can be omitted as it drops out when taking the derivative.

The FOC for consumption is given by

$$(1+\tau_{t+k}^c)\lambda_{t+k} P_{t+k} = V_{C,t+k}\ , \tag{B.3}$$

while the FOC for W_t^* is given by

$$\begin{aligned} 0 = \sum_{k=0}^{\infty} \beta^k E_t \Bigg[& U_{N,t+k} \frac{\partial N_{t+k}}{\partial W_t^*} \\ & + \lambda_{t+k} \left\{ (1-\varepsilon_w)(1-\tau_{t+k}^n) W_{t+k}^{\varepsilon_w} N_{t+k}^d \theta_w^k \left(\Gamma_{t,t+k}^{ind}\right)^{1-\varepsilon_w} (W_t^*)^{-\varepsilon_w} \right\} \Bigg] . \end{aligned} \tag{B.4}$$

Making use of

$$N_{t+k} \equiv \int_0^1 N_{t+k}^j dj = \int_0^1 \left(\frac{W_{t+k}^j}{W_{t+k}}\right)^{-\varepsilon_w} N_{t+k}^d \; dj = W_{t+k}^{\varepsilon_w} N_{t+k}^d \int_0^1 \left(W_{t+k}^j\right)^{-\varepsilon_w} dj$$
$$= W_{t+k}^{\varepsilon_w} N_{t+k}^d \left(\theta_w^k \left(\Gamma_{t,t+k}^{ind} W_t^*\right)^{-\varepsilon_w} + \left(1-\theta_w^k\right) X_{2,t+k}\right) \,, \tag{B.5}$$

we can evaluate the inner derivative in the first line of (B.4) to get

$$0 = \sum_{k=0}^{\infty} \beta^k E_t \Bigg[U_{N,t+k} \left(-\varepsilon^w\right) \frac{N_{t+k}^d}{W_{t+k}^{-\varepsilon^w}} \theta_w^k \left(\Gamma_{t,t+k}^{ind}\right)^{-\varepsilon_w} \left(W_t^*\right)^{-\varepsilon_w - 1}$$
$$+ \lambda_{t+k} \left\{ \left(1-\varepsilon_w\right) \left(1-\tau_{t+k}^n\right) W_{t+k}^{\varepsilon_w} N_{t+k}^d \theta_w^k \left(\Gamma_{t,t+k}^{ind}\right)^{1-\varepsilon_w} \left(W_t^*\right)^{-\varepsilon_w} \right\} \Bigg] \,. \tag{B.6}$$

Factoring out, and multiplying by $(W_t^*)^{-\varepsilon_w - 1}$ yields

$$0 = \sum_{k=0}^{\infty} \left(\beta\theta_w\right)^k E_t \lambda_{t+k} N_{t+k}^d W_{t+k}^{\varepsilon_w} \left(\Gamma_{t,t+k}^{ind}\right)^{-\varepsilon_w}$$
$$\times \left[\frac{U_{N,t+k}}{\lambda_{t+k}} \left(-\varepsilon_w\right) + \left(1-\tau_{t+k}^n\right) \left(1-\varepsilon_w\right) \Gamma_{t,t+k}^{ind} W_t^* \right] \tag{B.7}$$

or

$$0 = \sum_{k=0}^{\infty} \left(\beta\theta_w\right)^k E_t \lambda_{t+k} N_{t+k}^d W_{t+k}^{\varepsilon_w} \left(1-\tau_{t+k}^n\right) \left(\Gamma_{t,t+k}^{ind}\right)^{-\varepsilon_w}$$
$$\times \left[\frac{U_{N,t+k} \left(1+\tau_{t+k}^c\right) P_{t+k}}{V_{C,t+k} \left(1-\tau_{t+k}^n\right)} \left(-\varepsilon_w\right) + \left(1-\varepsilon_w\right) \Gamma_{t,t+k}^{ind} W_t^* \right] \,. \tag{B.8}$$

Using the after-tax MRS definition, this is equal to

$$0 = E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k V_{C,t+k} N_{t+k}^d W_{t+k}^{\varepsilon_w} \frac{1-\tau_{t+k}^n}{1+\tau_{t+k}^c} \left(\Gamma_{t,t+k}^{ind}\right)^{-\varepsilon_w} \left[MRS_{t+k} \frac{\varepsilon_w}{\varepsilon_w - 1} - \Gamma_{t,t+k}^{ind} \frac{W_t^*}{P_{t+k}} \right] \,. \tag{B.9}$$

Performing a log-linearization around the deterministic steady state yields

$$0 = \sum_{k=0}^{\infty} \left(\beta\theta_w\right)^k E_t \left[\frac{\varepsilon_w}{\varepsilon_w - 1} \times MRS\widehat{MRS}_{t+k} - \Gamma_k^{ind} \frac{W^*}{P} \left(\hat{W}_t^* - \hat{P}_{t+k} + \hat{\Gamma}_{t,t+k}^{ind}\right) \right] \tag{B.10}$$

or

$$\hat{W}_t^* = \left(1-\beta\theta_w\right) \sum_{k=0}^{\infty} \left(\beta\theta_w\right)^k E_t \left[\widehat{MRS}_{t+k} + \hat{P}_{t+k} - \hat{\Gamma}_{t,t+k}^{ind} \right] \,. \tag{B.11}$$

Note that compared to the EHL case, it is the economy-wide MRS that shows up here, not the individual one. Subtracting $\hat{W}_t$ from both sides and using (2.9), we can write this recursively as

$$\hat{W}_t^* - \hat{W}_t = -\beta\theta_w \hat{W}_t + \left(1-\beta\theta_w\right) \hat{\mu}_t^w \left(-1\right) + \beta\theta_w E_t \left(\hat{W}_{t+1}^* - \hat{\Gamma}_{t,t+1}^{ind}\right) \,. \tag{B.12}$$

Using (A.16) we obtain

$$\left(\frac{1}{1-\theta_w}\hat{W}_t - \frac{\theta_w}{1-\theta_w}\left(\hat{\Gamma}^{ind}_{t-1,t} + \hat{W}_{t-1}\right)\right) - \hat{W}_t = -\beta\theta_w \hat{W}_t + (1-\beta\theta_w)\,\hat{\mu}^w_t\,(-1)$$
$$+ \beta\theta_w E_t\left(\frac{1}{1-\theta_w}\hat{W}_{t+1} - \frac{\theta_w}{1-\theta_w}\left(\hat{\Gamma}^{ind}_{t,t+1} + \hat{W}_t\right) - \hat{\Gamma}^{ind}_{t,t+1}\right) \quad \text{(B.13)}$$

from which the New Keynesian Wage Phillips Curve follows as

$$\hat{\Pi}^w_t = \beta E_t \hat{\Pi}^w_{t+1} - \frac{(1-\beta\theta_w)(1-\theta_w)}{\theta_w}\hat{\mu}^w_t - \frac{\beta\theta_w}{1-\theta_w}E_t\hat{\Gamma}^{ind}_{t,t+1} + \frac{\theta_w}{1-\theta_w}\hat{\Gamma}^{ind}_{t-1,t}\,. \quad (3.4)$$

B.2 Rotemberg

The Lagrangian is

$$L = \sum_{k=0}^{\infty}\beta^k E_t \left[\begin{array}{l} U\left(C_{t+k}, N_{t+k}, \cdot\right) \\ -\lambda_{t+k}\left\{\begin{array}{l}\left(1+\tau^c_{t+k}\right)P_{t+k}C_{t+k} - \left(1-\tau^n_{t+k}\right)N^d_{t+k}\int_0^1 W^j_{t+k}\left(\frac{W^j_{t+k}}{W_{t+k}}\right)^{-\varepsilon_w} dj \\ +\frac{\phi_w}{2}\int_0^1\left(\frac{1}{\Gamma^{ind}_{t,t+k}}\frac{W^j_{t+k}}{W^j_{t+k-1}} - 1\right)^2 dj\,\Xi_{t+k} - X_{t+k}\end{array}\right\}\end{array}\right]. \quad \text{(B.14)}$$

The corresponding first order condition for the optimal wage is given by

$$\begin{aligned} 0 &= U_{N,t}\left(-\varepsilon_w\right)\int_0^1\left(\frac{W^j_t}{W_t}\right)^{-\varepsilon_w}\frac{N^d_t}{W^j_t}\,dj \\ &+ \lambda_t\left\{(1-\varepsilon_w)(1-\tau^n_t)N^d_t\int_0^1\left(\frac{W^j_t}{W_t}\right)^{-\varepsilon_w}dj - \phi_w\int_0^1\left(\frac{1}{\Gamma^{ind}_{t-1,t}}\frac{W^j_t}{W^j_{t-1}} - 1\right)\frac{1}{\Gamma^{ind}_{t-1,t}W^j_{t-1}}\,dj\;\Xi_t\right\} \\ &- E_t\lambda_{t+1}\left\{\phi_w\int_0^1\left(\frac{1}{\Gamma^{ind}_{t,t+1}}\frac{W^j_{t+1}}{W^j_t} - 1\right)(-1)\frac{W^j_{t+1}}{\Gamma^{ind}_{t,t+1}\left(W^j_t\right)^2}\,dj\;\Xi_{t+1}\right\}. \end{aligned} \quad \text{(B.15)}$$

Imposing symmetry

$$0 = U_N\left(C_t, N_t, \cdot\right)(-\varepsilon_w)\frac{N^d_t}{W_t} + \lambda_t\left\{(1-\varepsilon_w)(1-\tau^n_t)N^d_t - \phi_w\left(\frac{1}{\Gamma^{ind}_{t-1,t}}\frac{W_t}{W_{t-1}} - 1\right)\frac{\Xi_t}{\Gamma^{ind}_{t-1,t}W_{t-1}}\right\}$$
$$- E_t\lambda_{t+1}\left\{\phi_w\left(\frac{1}{\Gamma^{ind}_{t,t+1}}\frac{W_{t+1}}{W_t} - 1\right)(-1)\frac{W_{t+1}}{\Gamma^{ind}_{t,t+1}\left(W_t\right)^2}\Xi_{t+1}\right\}, \quad \text{(B.16)}$$

which is identical to equation (A.25).

C Elasticities of the after-tax MRS

C.1 Habits

C.1.1 Additively separable

First consider additively separable preferences with habits of the form

$$\frac{(C_t - \phi_c C_{t-1})^{1-\sigma} - 1}{1-\sigma} - \psi \frac{N^{1+\varphi}}{1+\varphi}\ , \tag{C.1}$$

where $0 \leq \phi_c \leq 1$ measures the degree of habits, $\varphi \geq 0$ is the inverse of the Frisch elasticity, $\sigma \geq 0$ determines the intertemporal elasticity of substitution, and $\psi > 0$ determines the weight of the disutility of labor.

If habits are internal, we get

$$V_{C_t} = (C_t - \phi_c C_{t-1})^{-\sigma} - \beta\phi_c \left(C_{t+1} - \phi_c C_t\right)^{-\sigma}$$

and in steady state

$$V_C = (1 - \beta\phi_c)\left((1-\phi_c)\,C\right)^{-\sigma}\ .$$

Similarly, the other partial derivatives are given by

$$\begin{aligned}
U_{N_t} &= -\psi N_t^{\varphi} \\
U_N &= -\psi N^{\varphi} \\
V_{C_tC_t} &= -\sigma(C_t - \phi_c C_{t-1})^{-\sigma-1} + \beta\phi_c^2\,(-\sigma)\left(C_{t+1} - \phi_c C_t\right)^{-\sigma-1} \\
V_{CC} &= \left(1 + \beta\phi_c^2\right)(-\sigma)\left((1-\phi_c)\,C\right)^{-\sigma-1} \\
V_{CN} &= 0
\end{aligned}$$

The marginal rate of substitution and its derivatives follow as

$$\begin{aligned}
MRS &= \frac{1+\tau^c}{1-\tau^n}\frac{\psi N^{\varphi}}{(1-\beta\phi_c)\left((1-\phi_c)\,C\right)^{-\sigma}} \\
MRS_N &= \varphi\frac{1+\tau^c}{1-\tau^n}\frac{\psi N^{\varphi-1}}{(1-\beta\phi_c)\left((1-\phi_c)\,C\right)^{-\sigma}} \\
MRS_C &= \frac{1+\tau^c}{1-\tau^n}\psi N^{\varphi}\,(-1)\,\frac{1}{(V_C)^2}V_{CC}
\end{aligned}$$

Therefore,

$$\varepsilon_n^{mrs} = \varphi \tag{C.2}$$

Table 6: Elasticities ε_n^{mrs},ε_c^{mrs}, and ε_{tot}^{mrs} for different felicity functions

	Add. separable	Add. sep. log leisure	Mult. separable	GHH
U	$\frac{C^{1-\sigma}-1}{1-\sigma} - \psi \frac{N^{1+\varphi}}{1+\varphi}$	$\frac{C^{1-\sigma}-1}{1-\sigma} + \psi \log(1-N_t)$	$\frac{\left(C^{\eta}(1-N)^{1-\eta}\right)^{1-\sigma}}{1-\sigma}$	$\frac{\left(C-\psi N^{1+\varphi}\right)^{1-\sigma}-1}{1-\sigma}$
V_N	$-\psi N^{\varphi}$	$-\psi \frac{1}{1-N}$	$-(1-\eta)(1-\sigma)\frac{U}{1-N}$	$\left(C-\psi N^{1+\varphi}\right)^{-\sigma} \times (-\psi)(1+\varphi)N^{\varphi}$
V_C	$C^{-\sigma}$	$C^{-\sigma}$	$\eta(1-\sigma)\frac{U}{C}$	$\left(C-\psi N^{1+\varphi}\right)^{-\sigma}$
V_{CC}	$-\sigma C^{-\sigma-1}$	$-\sigma C^{-\sigma-1}$	$\eta(1-\sigma)(\eta(1-\sigma)-1)\frac{U}{C^2}$	$-\sigma\left(C-\psi N^{1+\varphi}\right)^{-\sigma-1}$
V_{CN}	0	0	$\eta(1-\sigma)(1-\eta)(\sigma-1) \times \frac{U}{C(1-N)}$	$\sigma\left(C-\psi N^{1+\varphi}\right)^{-\sigma-1} \times \psi(1+\varphi)N^{\varphi}$
MRS	$\frac{1+\tau^c}{1-\tau^n}\frac{\psi N^{\varphi}}{C^{-\sigma}}$	$\frac{1+\tau^c}{1-\tau^n}\frac{\psi(1-N_t)^{-1}}{C^{-\sigma}}$	$\frac{1+\tau^c}{1-\tau^n}\frac{1-\eta}{\eta}\frac{C}{1-N}$	$\frac{1+\tau^c}{1-\tau^n}\psi(1+\varphi)N^{\varphi}$
MRS_N	$\varphi\frac{1+\tau^c}{1-\tau^n}\frac{\psi N^{\varphi-1}}{C^{-\sigma}}$	$\frac{1+\tau^c}{1-\tau^n}\frac{\psi(1-N_t)^{-2}}{C^{-\sigma}}$	$\frac{1+\tau^c}{1-\tau^n}\frac{1-\eta}{\eta}\frac{C}{(1-N)^2}$	$\frac{1+\tau^c}{1-\tau^n}\psi(1+\varphi)\varphi N^{\varphi-1}$
MRS_C	$\frac{1+\tau^c}{1-\tau^n}\sigma\frac{\psi N^{\varphi}}{C^{1-\sigma}}$	$\frac{1+\tau^c}{1-\tau^n}\sigma\frac{\psi(1-N)^{-1}}{C^{1-\sigma}}$	$\frac{1+\tau^c}{1-\tau^n}\frac{1-\eta}{\eta}\frac{(1-\phi_c)}{(1-N)}$	0
ε_n^{mrs}	φ	$\frac{N}{1-N}$	$\frac{N}{1-N}$	φ
ε_c^{mrs}	σ	σ	1	0
ε_{tot}^{mrs}	φ	$\frac{N}{1-N}$	$\left[1-\frac{(1-\eta)(\sigma-1)}{\eta(1-\sigma)-1}\right]\frac{N}{1-N}$	φ
ε_{tot}^{mrs} (int. habits)	φ	$\frac{N}{1-N}$	$\left[1-\frac{(1-\eta)(\sigma-1)}{\eta(1-\sigma)-1}\frac{(1-\phi_c)}{(1+\phi_c^2\beta)}\right]\frac{N}{1-N}$	φ
ε_{tot}^{mrs} (ext. habits)	φ	$\frac{N}{1-N}$	$\left[1-\frac{(1-\eta)(\sigma-1)}{\eta(1-\sigma)-1}(1-\phi_c)\right]\frac{N}{1-N}$	φ

Notes: Elasticities of the after-tax marginal rate of substitution with respect to hours worked, ε_n^{mrs}, with respect to consumption, ε_c^{mrs}, and the total elasticity, ε_{tot}^{mrs}, for additively separable preferences in consumption and hours worked (first column), additively separable preferences in consumption and log leisure (second column), for multiplicative preferences (third column), and Greenwood, Hercowitz, and Huffman (1988)-type preferences (fourth column). The last two rows display the total elasticity when internal or external habits in consumption of the form $C_t - \phi_c C_{t-1}$ are assumed.

and

$$\varepsilon_c^{mrs} = \frac{\frac{1+\tau^c}{1-\tau^n}(-U_N)\frac{-1}{(V_C)^2}V_{CC}}{\frac{1+\tau^c}{1-\tau^n}\frac{U_N}{V_C}}C = (-1)\,\frac{V_{CC}C}{V_C} = (-1)\,\frac{(1+\beta\phi_c^2)\,(-\sigma)\,((1-\phi_c)\,C)^{-\sigma-1}C}{(1-\beta\phi_c)\,((1-\phi_c)\,C)^{-\sigma}}$$
$$= \frac{1+\beta\phi_c^2}{1-\beta\phi_c}\frac{\sigma}{(1-\phi_c)}\,, \tag{C.3}$$

Because of $V_{CN} = 0$, we also have[24]

$$\varepsilon_{tot}^{mrs} = \varepsilon_n^{mrs}\,, \tag{C.6}$$

If habits are external, we get the partial derivatives

$$\begin{aligned}
V_{N_t} &= -\psi N_t^{\varphi} \\
V_N &= -\psi N^{\varphi} \\
V_{C_t} &= (C_t - \phi_c C_{t-1})^{-\sigma} \\
V_C &= ((1-\phi_c)\,C)^{-\sigma} \\
V_{C_tC_t} &= (-\sigma)\,(C_t - \phi_c C_{t-1})^{-\sigma-1} \\
V_{CC} &= (-\sigma)\,((1-\phi_c)\,C)^{-\sigma-1} \\
V_{CN} &= 0
\end{aligned}$$

and the marginal rate of substitution

$$\begin{aligned}
MRS &= \frac{1+\tau^c}{1-\tau^n}\frac{\psi N^{\varphi}}{((1-\phi_c)\,C)^{-\sigma}} \\
MRS_N &= \varphi\frac{1+\tau^c}{1-\tau^n}\frac{\psi N^{\phi-1}}{((1-\phi_c)\,C)^{-\sigma}} \\
MRS_C &= \frac{1+\tau^c}{1-\tau^n}\frac{\psi N^{\varphi}}{((1-\phi_c)\,C)^{-\sigma+1}}
\end{aligned}$$

and therefore

$$\varepsilon_c^{mrs} = (-1)\,\frac{V_{CC}C}{V_C} = (-1)\,\frac{(-\sigma)((1-\phi_c)\,C)^{-\sigma-1}C}{((1-\phi_c)\,C)^{-\sigma}} = \frac{\sigma}{(1-\phi_c)} \tag{C.7}$$

with similar expressions for log leisure. As a consequence, ε_n^{mrs} is the same as in the case

[24]A related functional form with unitary Frisch elasticity considers log utility in leisure:

$$\frac{(C_t - \phi_c C_{t-1})^{1-\sigma} - 1}{1-\sigma} - \psi\log(1-N)\,. \tag{C.4}$$

and yields

$$\varepsilon_{tot}^{mrs} = \varepsilon_n^{mrs} = N/(1-N) \tag{C.5}$$

.

of internal habits and, because of $V_{CN} = 0$, we also have

$$\varepsilon_{tot}^{mrs} = \varepsilon_{n}^{mrs} \,. \tag{C.8}$$

C.1.2 Multiplicatively separable

Consider a multiplicative felicity function[25] with habits

$$U_t = \frac{\left((C_t - \phi_c C_{t-1})^{\eta} (1-N)^{1-\eta}\right)^{1-\sigma}}{1-\sigma} = \frac{(C_t - \phi_c C_{t-1})^{\eta(1-\sigma)} (1-N)^{(1-\eta)(1-\sigma)}}{1-\sigma} \,, \tag{C.9}$$

where $0 \leq \phi_c \leq 1$ measures the degree of habits, $0 \leq \eta \leq 1$ determines the weight of leisure, and $\sigma \geq 0$ determines the intertemporal elasticity of substitution.

If habits are internal, we have

$$\begin{aligned}
V_{N_t} &= (1-\eta)\,(C_t - \phi_c C_{t-1})^{\eta(1-\sigma)}\,(-1)\,(1-N_t)^{(1-\eta)(1-\sigma)-1} \\
&= -\,(1-\eta)\,(1-\sigma)\,\frac{U_t}{(1-N_t)} \\
V_N &= -\,(1-\eta)\,((1-\phi_c)\,C)^{\eta(1-\sigma)}(1-N)^{(1-\eta)(1-\sigma)-1} \\
&= -\,(1-\eta)\,(1-\sigma)\,\frac{U}{(1-N)} \\
V_{C_t} &= \eta\,(C_t - \phi_c C_{t-1})^{\eta(1-\sigma)-1}\,(1-N_t)^{(1-\eta)(1-\sigma)} \\
&\quad - \phi_c \beta \eta\,(C_{t+1} - \phi_c C_t)^{\eta(1-\sigma)-1}\,(1-N_{t+1})^{(1-\eta)(1-\sigma)} \\
&= \eta\,(1-\sigma)\left(\frac{U_t}{C_t - \phi_c C_{t-1}} - \beta\phi_c \frac{U_{t+1}}{C_{t+1} - \phi_c C_t}\right) \\
V_C &= \eta\,(1-\phi_c\beta)\,((1-\phi_c)\,C)^{\eta(1-\sigma)-1}\,(1-N_t)^{(1-\eta)(1-\sigma)} \\
&= \eta\,(1-\sigma)\,(1-\phi_c\beta)\,\frac{U}{(1-\phi_c)C} \\
V_{C_tC_t} &= \eta\,(\eta\,(1-\sigma)-1)\,(C_t - \phi_c C_{t-1})^{\eta(1-\sigma)-2}(1-N_t)^{(1-\eta)(1-\sigma)} \\
&\quad - \phi_c\beta\eta\,(\eta\,(1-\sigma)-1)\,(-\phi_c)\,(C_{t+1} - \phi_c C_t)^{\eta(1-\sigma)-2}(1-N_{t+1})^{(1-\eta)(1-\sigma)} \\
V_{CC} &= \eta\,(\eta\,(1-\sigma)-1)\left(1+\phi_c^2\beta\right)((1-\phi_c)\,C)^{\eta(1-\sigma)-2}(1-N_t)^{(1-\eta)(1-\sigma)} \\
&= \frac{(\eta\,(1-\sigma))\,(\eta\,(1-\sigma)-1)\,(1+\phi_c^2\beta)\,U}{((1-\phi_c)C)^2} \\
V_{C_tN_t} &= \eta\,(C_t - \phi_c C_{t-1})^{\eta(1-\sigma)-1}\,(1-\eta)\,(1-\sigma)\,(-1)\,(1-N_t)^{(1-\eta)(1-\sigma)-1} \\
V_{CN} &= \eta\,((1-\phi_c)\,C)^{\eta(1-\sigma)-1}\,(1-\eta)\,(\sigma-1)\,(1-N)^{(1-\eta)(1-\sigma)-1} \\
&= (\eta\,(1-\sigma))\,(1-\eta)\,(\sigma-1)\,\frac{U}{(1-\phi_c)C\,(1-N)}
\end{aligned}$$

[25]It has e.g. been used by Backus et al. (1992).

Therefore,

$$
\begin{aligned}
MRS &= \frac{1+\tau^c}{1-\tau^n}\frac{1-\eta}{\eta}\frac{(1-\phi_c)C}{(1-\phi_c\beta)\,(1-N)} \\
MRS_N &= \frac{1+\tau^c}{1-\tau^n}\frac{1-\eta}{\eta}\frac{(1-\phi_c)C}{(1-\phi_c\beta)\,(1-N)^2} \\
MRS_C &= \frac{1+\tau^c}{1-\tau^n}\frac{1-\eta}{\eta}\frac{1-\phi_c}{(1-\phi_c\beta)\,(1-N)}
\end{aligned}
$$

and

$$
\varepsilon_n^{mrs} = \frac{\frac{1+\tau^c}{1-\tau^n}\frac{1-\eta}{\eta}\frac{(1-\phi_c)C}{(1-\phi_c\beta)(1-N)^2}N}{\frac{1+\tau^c}{1-\tau^n}\frac{1-\eta}{\eta}\frac{(1-\phi_c)C}{(1-\phi_c\beta)(1-N)}} = \frac{N}{1-N} \tag{C.10}
$$

$$
\varepsilon_c^{mrs} = \frac{\frac{1+\tau^c}{1-\tau^n}\frac{1-\eta}{\eta}\frac{1-\phi_c}{1-\phi_c\beta}\frac{1}{1-N}}{\frac{1+\tau^c}{1-\tau^n}\frac{1-\eta}{\eta}\frac{1-\phi_c}{1-\phi_c\beta}\frac{C}{1-N}}C = 1 \tag{C.11}
$$

Finally

$$
\begin{aligned}
\frac{V_{CN}}{V_{CC}} &= \frac{(\eta\,(1-\sigma))\,(1-\eta)\,(\sigma-1)\,\frac{U}{(1-\phi_c)C(1-N)}}{\frac{(\eta(1-\sigma))(\eta(1-\sigma)-1)(1+\phi_c^2\beta)U}{((1-\phi_c)C)^2}} \\
&= \frac{(1-\eta)\,(\sigma-1)}{\eta(1-\sigma)-1}\frac{(1-\phi_c)}{(1+\phi_c^2\beta)}\frac{C}{1-N}
\end{aligned}
$$

and

$$
\begin{aligned}
\varepsilon_{tot}^{mrs} &= -\frac{V_{CN}N}{V_{CC}C}\varepsilon_c^{mrs} + \varepsilon_n^{mrs} \\
&= -\frac{(1-\eta)\,(\sigma-1)}{\eta(1-\sigma)-1}\frac{(1-\phi_c)}{(1+\phi_c^2\beta)}\frac{CN}{(1-N)C} \times 1 + \frac{N}{1-N} \\
&= \left[1-\frac{(1-\eta)\,(\sigma-1)}{\eta(1-\sigma)-1}\frac{(1-\phi_c)}{(1+\phi_c^2\beta)}\right]\frac{N}{1-N}
\end{aligned} \tag{C.12}
$$

In case of $\sigma = 1$, i.e. log utility, utility becomes separable again and (C.12) reduces to (C.5).

With external habits,

$$\begin{aligned}
V_{C_t} &= \eta \left(C_t - \phi_c C_{t-1}\right)^{\eta(1-\sigma)-1} \left(1 - N_t\right)^{(1-\eta)(1-\sigma)} \\
&= \eta \left(1 - \sigma\right) \frac{U_t}{C_t - \phi_c C_{t-1}} \\
V_C &= \eta \left(\left(1 - \phi_c\right) C\right)^{\eta(1-\sigma)-1} \left(1 - N_t\right)^{(1-\eta)(1-\sigma)} \\
&= \eta \left(1 - \sigma\right) \frac{U}{(1 - \phi_c) C} \\
V_{C_t C_t} &= \eta \left(\eta \left(1 - \sigma\right) - 1\right) \left(C_t - \phi_c C_{t-1}\right)^{\eta(1-\sigma)-2} \left(1 - N_t\right)^{(1-\eta)(1-\sigma)} \\
V_{CC} &= \eta \left(\eta \left(1 - \sigma\right) - 1\right) \left(\left(1 - \phi_c\right) C\right)^{\eta(1-\sigma)-2} (1 - N)^{(1-\eta)(1-\sigma)} \\
&= \frac{\left(\eta \left(1 - \sigma\right)\right) \left(\eta \left(1 - \sigma\right) - 1\right) U}{((1 - \phi_c) C)^2} \\
V_{C_t N_t} &= \eta \left(C_t - \phi_c C_{t-1}\right)^{\eta(1-\sigma)-1} \left(1 - \eta\right) \left(1 - \sigma\right) \left(-1\right) \left(1 - N_t\right)^{(1-\eta)(1-\sigma)-1} \\
V_{CN} &= \eta \left(\left(1 - \phi_c\right) C\right)^{\eta(1-\sigma)-1} \left(1 - \eta\right) \left(\sigma - 1\right) \left(1 - N\right)^{(1-\eta)(1-\sigma)-1} \\
&= \left(\eta \left(1 - \sigma\right)\right) \left(1 - \eta\right) \left(\sigma - 1\right) \frac{U}{(1 - \phi_c) C \left(1 - N\right)} \ .
\end{aligned}$$

Therefore,

$$\begin{aligned}
MRS &= \frac{1 + \tau^c}{1 - \tau^n} \frac{1 - \eta}{\eta} \frac{(1 - \phi_c) C}{1 - N} \\
MRS_N &= \frac{1 + \tau^c}{1 - \tau^n} \frac{1 - \eta}{\eta} \frac{(1 - \phi_c) C}{1 - N} \\
MRS_C &= \frac{1 + \tau^c}{1 - \tau^n} \frac{1 - \eta}{\eta} \frac{1 - \phi_c}{(1 - N)}
\end{aligned}$$

and

$$\varepsilon_n^{mrs} = \frac{\frac{1+\tau^c}{1-\tau^n} \frac{1-\eta}{\eta} \frac{(1-\phi_c) C}{(1-N)^2} N}{\frac{1+\tau^c}{1-\tau^n} \frac{1-\eta}{\eta} \frac{(1-\phi_c) C}{(1-N)}} = \frac{N}{1 - N} \tag{C.13}$$

$$\varepsilon_c^{mrs} = \frac{\frac{1+\tau^c}{1-\tau^n} \frac{1-\eta}{\eta} \frac{1-\phi_c}{1-N}}{\frac{1+\tau^c}{1-\tau^n} \frac{1-\eta}{\eta} \frac{(1-\phi_c) C}{1-N}} C = 1 \tag{C.14}$$

Finally

$$\begin{aligned}
\frac{V_{CN}}{V_{CC}} &= \frac{\left(\eta \left(1 - \sigma\right)\right) \left(1 - \eta\right) \left(\sigma - 1\right) \frac{U}{(1-\phi_c) C (1-N)}}{\frac{(\eta(1-\sigma))(\eta(1-\sigma)-1) U}{((1-\phi_c) C)^2}} \\
&= \frac{\left(1 - \eta\right) \left(\sigma - 1\right)}{\eta \left(1 - \sigma\right) - 1} \frac{(1 - \phi_c) C}{1 - N}
\end{aligned}$$

$$
\begin{aligned}
\varepsilon^{mrs}_{tot} &= -\frac{V_{CN}N}{V_{CC}C}\varepsilon^{mrs}_{c} + \varepsilon^{mrs}_{n} \\
&= -\frac{(1-\eta)\,(\sigma-1)}{\eta(1-\sigma)-1}(1-\phi_c)\frac{CN}{(1-N)C}\times 1 + \frac{N}{1-N} \\
&= \left[1 - \frac{(1-\eta)\,(\sigma-1)}{\eta(1-\sigma)-1}(1-\phi_c)\right]\frac{N}{1-N}
\end{aligned}
\tag{C.15}
$$

C.1.3 GHH

Consider GHH preferences with habits of the form

$$
U = \frac{\left(C_t - \phi_c C_{t-1} - \psi N_t^{1+\varphi}\right)^{1-\sigma} - 1}{1-\sigma} - 1\ , \tag{C.16}
$$

where $0 \leq \phi_c \leq 1$ measures the degree of habits, $\varphi \geq 0$ is related to the Frisch elasticity, $\sigma \geq 0$ determines the intertemporal elasticity of substitution ($\sigma = 1$ corresponds to log utility), and $\psi > 0$ determines weight of the disutility of labor. In case of internal habits we get

$$
\begin{aligned}
V_{N_t} &= \left(C_t - \phi_c C_{t-1} - \psi N_t^{1+\varphi}\right)^{-\sigma}(-\psi)\,(1+\varphi)N_t^{\varphi} \\
V_N &= \left((1-\phi_c)C - \psi N^{1+\varphi}\right)^{-\sigma}(-\psi)\,(1+\varphi)N^{\varphi} \\
V_{C_t} &= \left(C_t - \phi_c C_{t-1} - \psi N_t^{1+\varphi}\right)^{-\sigma} - \beta\phi_c\left(C_{t+1} - \phi_c C_t - \psi N_{t+1}^{1+\varphi}\right)^{-\sigma} \\
V_C &= (1-\beta\phi_c)\left((1-\phi_c)C - \psi N^{1+\varphi}\right)^{-\sigma} \\
V_{C_tC_t} &= -\sigma\Big(C_t - \phi_c C_{t-1} - \psi N_t^{1+\varphi}\Big)^{-\sigma-1} - \beta\phi_c\,(-\sigma)\,(-\phi_c)\left(C_{t+1} - \phi_c C_t - \psi N_{t+1}^{1+\varphi}\right)^{-\sigma-1} \\
V_{CC} &= -\sigma\left(1+\beta\phi_c^2\right)\left((1-\phi_c)C - \psi N^{1+\varphi}\right)^{-\sigma-1} \\
V_{C_tN_t} &= -\sigma\Big(C_t - \phi_c C_{t-1} - \psi N_t^{1+\varphi}\Big)^{-\sigma-1}(-\psi)\,(1+\varphi)\,N_t^{\varphi} \\
V_{CN} &= \sigma\Big((1-\phi_c)\,C - \psi N^{1+\varphi}\Big)^{-\sigma-1}\psi\,(1+\varphi)\,N^{\varphi}
\end{aligned}
$$

and therefore

$$
\begin{aligned}
MRS &= \frac{1+\tau^c}{1-\tau^n}\frac{\left((1-\phi_c)C - \psi N^{1+\varphi}\right)^{-\sigma}\psi(1+\varphi)N^{\varphi}}{(1-\beta\phi_c)\left((1-\phi_c)C - \psi N^{1+\varphi}\right)^{-\sigma}} = \frac{1+\tau^c}{1-\tau^n}\frac{\psi(1+\varphi)N^{\varphi}}{(1-\beta\phi_c)} \\
MRS_N &= \frac{1+\tau^c}{1-\tau^n}\psi(1+\varphi)\varphi\frac{N^{\varphi-1}}{(1-\beta\phi_c)} \\
MRS_C &= 0
\end{aligned}
$$

and

$$\varepsilon_n^{mrs} = \frac{\frac{1+\tau^c}{1-\tau^n}\frac{\psi(1+\varphi)\varphi}{(1-\beta\phi_c)}N^{\varphi-1}N}{\frac{1+\tau^c}{1-\tau^n}\frac{\psi(1+\varphi)}{(1-\beta\phi_c)}N^{\varphi}} = \varphi \tag{C.17}$$

$$\varepsilon_c^{mrs} = 0 \tag{C.18}$$

and therefore

$$\varepsilon_{tot}^{mrs} = \varepsilon_n^{mrs}\ . \tag{C.19}$$

For external habits

$$\begin{aligned}
V_{N_t} &= \left(C_t - \phi_c C_{t-1} - \psi N_t^{1+\varphi}\right)^{-\sigma}(-\psi)(1+\varphi)N_t^{\varphi} \\
V_N &= \left((1-\phi_c)C - \psi N^{1+\varphi}\right)^{-\sigma}(-\psi)(1+\varphi)N^{\varphi} \\
V_{C_t} &= \left(C_t - \phi_c C_{t-1} - \psi N_t^{1+\varphi}\right)^{-\sigma} \\
V_C &= \left((1-\phi_c)C - \psi N^{1+\varphi}\right)^{-\sigma} \\
V_{C_tC_t} &= -\sigma\left(C_t - \phi_c C_{t-1} - \psi N_t^{1+\varphi}\right)^{-\sigma-1} \\
V_{CC} &= -\sigma\left((1-\phi_c)C - \psi N^{1+\varphi}\right)^{-\sigma-1} \\
V_{C_tN_t} &= -\sigma\left(C_t - \phi_c C_{t-1} - \psi N_t^{1+\varphi}\right)^{-\sigma-1}(-\psi)(1+\varphi)N_t^{\varphi} \\
V_{CN} &= \sigma\left((1-\phi_c)C - \psi N^{1+\varphi}\right)^{-\sigma-1}\psi(1+\varphi)N^{\varphi}
\end{aligned}$$

and therefore

$$\begin{aligned}
MRS &= \frac{1+\tau^c}{1-\tau^n}\frac{\left((1-\phi_c)C - \psi N^{1+\varphi}\right)^{-\sigma}\psi(1+\varphi)N^{\varphi}}{\left((1-\phi_c)C - \psi N^{1+\varphi}\right)^{-\sigma}} = \frac{1+\tau^c}{1-\tau^n}\psi(1+\varphi)N^{\varphi} \\
MRS_N &= \frac{1+\tau^c}{1-\tau^n}\psi(1+\varphi)\varphi N^{\varphi-1} \\
MRS_C &= 0
\end{aligned}$$

and

$$\varepsilon_n^{mrs} = \frac{\frac{1+\tau^c}{1-\tau^n}\psi(1+\varphi)\varphi N^{\varphi-1}N}{\frac{1+\tau^c}{1-\tau^n}\psi(1+\varphi)N^{\varphi}} = \varphi \tag{C.20}$$

$$\varepsilon_c^{mrs} = 0 \tag{C.21}$$

and therefore

$$\varepsilon_{tot}^{mrs} = \varepsilon_n^{mrs}\ . \tag{C.22}$$

D Welfare

To keep the exposition simple, we in the following abstract from sticky prices. As long as price dispersion/price adjustment costs are 0 in steady state, they could easily be added without affecting the conclusions derived.[26] Without loss of generality, we also omit the preference shocks for notational brevity.

Table 7: Model moments from the Galí (2015), Chapter 6 model

	Strict Targeting			Flexible Targeting		
	Price	Wage	Comp.	Price	Wage	Comp.
	Technology Shock					
$\sigma(\pi_p)$	0.000	0.135	0.123	0.298	0.243	0.246
$\sigma(\pi_w)$	0.266	0.000	0.021	0.238	0.165	0.169
$\sigma(\tilde{y})$	3.417	0.204	0.000	0.848	1.183	1.113
	Demand Shock					
$\sigma(\pi_p)$	0.000	0.000	0.000	0.026	0.041	0.038
$\sigma(\pi_w)$	0.000	0.000	0.000	0.054	0.066	0.064
$\sigma(\tilde{y})$	0.000	0.000	0.000	1.082	1.054	1.061

Notes: Displayed are the variance of log price inflation π_p, log wage inflation π_w, and of the log output gap $\tilde{y}$. Numbers have been multiplied by 100.

D.1 SGU framework

In the SGU framework, household members supply the same homogenous labor good to unions so that the aggregate utility function is given by

$$U^{SGU} = \int_0^1 U_t\left(j\right) dj = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \tag{D.1}$$

A second-order Taylor approximation around the deterministic steady state yields

$$\hat{U}^{SGU} = C^{-\sigma}\hat{C}_t - \frac{1}{2}C^{-\sigma-1}\sigma\hat{C}_t^2 - N^{\varphi}\hat{N}_t - \frac{1}{2}N^{\varphi-1}\varphi\hat{N}_t^2 \,. \tag{D.2}$$

[26] In case nominal rigidities affect the deterministic steady state, there would be interaction effects between price and wage rigidities.

D.1.1 Rotemberg

In a symmetric Rotemberg equilibrium, we can drop all indices j. The aggregate production function and the resource constraint are

$$Y_t = A_t N_t^{1-\alpha} \tag{D.3}$$

$$Y_t = C_t + \frac{\phi_w^{SGU}}{2} (\Pi_t^w - 1)^2 Y_t \, . \tag{D.4}$$

We can use them to express consumption as a function of production and wage adjustment costs:

$$C_t = A_t N_t^{1-\alpha} \left(1 - \frac{\phi_w^{SGU}}{2} (\Pi_t^w - 1)^2\right) \tag{D.5}$$

A second-order Taylor expansion yields

$$\begin{aligned} C_t - C =& N^{1-\alpha} \left(1 - \frac{\phi_w^{SGU}}{2} (\Pi^w - 1)^2\right) (A_t - A) + (1-\alpha) A N^{-\alpha} \left(1 - \frac{\phi_w^{SGU}}{2} (\Pi^w - 1)^2\right) (N_t - N) \\ & - A N^{1-\alpha} \phi_w^{SGU} (\Pi^w - 1) (\Pi_t^w - \Pi^w) \\ & + \frac{1}{2} 2 (1-\alpha) N^{-\alpha} \left(1 - \frac{\phi_w^{SGU}}{2} (\Pi^w - 1)^2\right) (N_t - N) (A_t - A) - \frac{1}{2} A N^{1-\alpha} \phi_w^{SGU} (\Pi_t^w - \Pi^w)^2 \\ & + \frac{1}{2} (1-\alpha)(-\alpha) A N^{-\alpha-1} \left(1 - \frac{\phi_w^{SGU}}{2} (\Pi^w - 1)^2\right) (N_t - N)^2 \, . \end{aligned} \tag{D.6}$$

Letting hats denote deviations from steady state, $\hat{X}_t = X_t - X$, and imposing a zero inflation steady state, we can write hours worked as

$$\begin{aligned} \hat{N}_t =& \frac{1}{(1-\alpha)} N^{-\alpha} \Big[\hat{C}_t - N^{1-\alpha} \hat{A}_t - (1-\alpha) N^{-\alpha} \hat{N}_t - (1-\alpha) N^{-\alpha} \hat{N}_t \hat{A}_t \\ & + \frac{1}{2} \alpha (1-\alpha) N^{-\alpha-1} \hat{N}_t^2 + \frac{\phi_w^{SGU}}{2} C \left(\hat{\Pi}_t^w\right)^2 \Big] \, . \end{aligned} \tag{D.7}$$

Plugging into (D.2) yields

$$
\begin{aligned}
\hat{U}^{SGU}_{Rotemberg} =& C^{-\sigma}\hat{C}_t - \frac{1}{2}C^{-\sigma-1}\sigma\hat{C}_t^2 - N^{\varphi}\hat{N}_t - \frac{1}{2}N^{\varphi-1}\varphi\hat{N}_t^2 \\
=& C^{-\sigma}\hat{C}_t - \frac{1}{2}C^{-\sigma-1}\sigma\hat{C}_t^2 \\
& - N^{\varphi}\left(\frac{1}{(1-\alpha)\,N^{-\alpha}}\begin{bmatrix}\hat{C}_t - N^{1-\alpha}\hat{A}_t - (1-\alpha)\,N^{-\alpha}\hat{N}_t - (1-\alpha)\,N^{-\alpha}\hat{N}_t\hat{A}_t \\ +\frac{1}{2}\alpha\,(1-\alpha)\,N^{-\alpha-1}\hat{N}_t^2 + \frac{\phi_w^{SGU}}{2}C\left(\hat{\Pi}_t^w\right)^2\end{bmatrix}\right) \\
& - \frac{1}{2}N^{\varphi-1}\varphi\hat{N}_t^2 \\
=& \left(C^{-\sigma} - \frac{N^{\varphi}}{(1-\alpha)\,N^{-\alpha}}\right)\hat{C}_t - \frac{1}{2}C^{-\sigma-1}\sigma\hat{C}_t^2 \\
& - \left(\frac{1}{2}N^{\varphi-1}\varphi - \frac{N^{\varphi}}{(1-\alpha)\,N^{-\alpha}}\frac{1}{2}\alpha\,(1-\alpha)\,N^{-\alpha-1}\right)\hat{N}_t^2 \\
& + \frac{N^{\varphi}N^{\alpha}}{(1-\alpha)\,N^{-\alpha}}\hat{A}_t + N^{\varphi}\hat{N}_t\hat{A}_t - \frac{N^{\varphi}}{(1-\alpha)\,N^{-\alpha}}C\frac{\phi_w^{SGU}}{2}\left(\hat{\Pi}_t^w\right)^2 .
\end{aligned}
\tag{D.8}
$$

Note that in an efficient steady state, the linear term in consumption will drop out as

$$
C^{-\sigma} - \frac{N^{\varphi}}{(1-\alpha)\,N^{-\alpha}} = 0 . \tag{D.9}
$$

This is nothing else than the condition that the marginal rate of substitution is equal to the marginal product of labor. This is the well-known result that with an efficient steady state, a linear approximation to the policy functions is sufficient to get a second order accurate welfare measure. Second order terms from the policy functions plugged into the second order approximated utility function would result in terms of order higher than two.

D.1.2 Calvo

In the symmetric Calvo equilibrium, aggregate output and the resource constraint are given by

$$
C_t = Y_t = A_t\left(\frac{N_t}{S_t^w}\right)^{1-\alpha} , \tag{D.10}
$$

where we aggregated over labor services:

$$
N_t \equiv \int_0^1 N_t^j dj = \int_0^1 \left(\frac{W_t^j}{W_t}\right)^{-\varepsilon_w} N_t^d dj = N_t^d \int_0^1 \left(\frac{W_t^j}{W_t}\right)^{-\varepsilon_w} dj . \tag{D.11}
$$

Defining the auxiliary variable $S_t^W \equiv \int_0^1 \left(\frac{W_t^j}{W_t}\right)^{-\varepsilon_w} dj$, which captures wage dispersion, implies

$$
N_t^d = \frac{N_t}{S_t^W} . \tag{D.12}
$$

A second order approximation to (D.10) yields

$$
\begin{aligned}
\hat{C}_t =& \left(\frac{N}{S^w}\right)^{1-\alpha}(A_t - A) + \frac{(1-\alpha)\, AN^{-\alpha}}{(S^w)^{1-\alpha}}(N_t - N) - (1-\alpha)\, AN^{1-\alpha}(S^w)^{-(1-\alpha)-1}(S_t^w - S^w) \\
&+ \frac{1}{2}2\begin{bmatrix} \frac{(1-\alpha)\, N^{-\alpha}}{(S^w)^{1-\alpha}}(N_t - N)(A_t - A) - N^{1-\alpha}(S^w)^{-(1-\alpha)-1}(S_t^w - S^w)(A_t - A) \\ -(1-\alpha)(-(1-\alpha))\, AN^{-\alpha}(S^w)^{-(1-\alpha)-1}(S_t^w - S^w)(N_t - N) \end{bmatrix} \\
&+ \frac{1}{2}\begin{pmatrix} AN^{1-\alpha}(-(1-\alpha)(-(1-\alpha)-1))(S^w)^{-(1-\alpha)-2}(S_t^w - S^w)^2 \\ +(1-\alpha)(-\alpha)\, A\frac{N^{-\alpha-1}}{(S^w)^{1-\alpha}}(N_t - N)^2 \end{pmatrix} \\
&+ \left[(1-\alpha)\, N^{-\alpha}\hat{N}_t\hat{A}_t - N^{1-\alpha}\hat{S}_t^w\hat{A}_t - (1-\alpha)(-(1-\alpha))\, N^{-\alpha}\hat{S}_t^w\hat{N}_t\right] \\
&- (-(1-\alpha)(-(1-\alpha)-1))\, N^{1-\alpha}\left(\hat{S}_t^w\right)^2 + \frac{1}{2}\alpha(\alpha-1)\, N^{\alpha-2}\hat{N}_t^2\,,
\end{aligned}
\tag{D.13}
$$

where the second equality uses that in steady state $A = 1$ and $S_t^w = 1$. We will show below that $\hat{S}_t^w = 0$ up to first order, so that

$$
\hat{N}_t = \frac{1}{(1-\alpha)\, N^{-\alpha}}\begin{bmatrix} \hat{C}_t + (1-\alpha)\, C\hat{S}_t^w - N^{1-\alpha}\hat{A}_t - (1-\alpha)\, N^{-\alpha}\hat{N}_t \\ -(1-\alpha)\, N^{-\alpha}\hat{N}_t\hat{A}_t + \frac{1}{2}\alpha(1-\alpha)\, N^{-\alpha-1}\hat{N}_t^2 \end{bmatrix}. \tag{D.14}
$$

Plugging this into the approximated felicity function (D.2) yields

$$
\begin{aligned}
\hat{U}_{Calvo}^{SGU} =& C^{-\sigma}\hat{C}_t - \frac{1}{2}C^{-\sigma-1}\sigma\hat{C}_t^2 - N^{\varphi}\left(\frac{1}{(1-\alpha)\, N^{-\alpha}}\begin{bmatrix} \hat{C}_t + (1-\alpha)C\hat{S}_t^w - N^{1-\alpha}\hat{A}_t - (1-\alpha)\, N^{-\alpha}\hat{N}_t \\ -(1-\alpha)\, N^{-\alpha}\hat{N}_t\hat{A}_t + \frac{1}{2}\alpha(1-\alpha)\, N^{-\alpha-1}\hat{N}_t^2 \end{bmatrix}\right) \\
&- \frac{1}{2}N^{\varphi-1}\varphi\hat{N}_t^2 \\
=& \left(C^{-\sigma} - \frac{N^{\varphi}}{(1-\alpha)\, N^{-\alpha}}\right)\hat{C}_t - \frac{1}{2}C^{-\sigma-1}\sigma\hat{C}_t^2 \\
&- \left(\frac{1}{2}N^{\varphi-1}\varphi - \frac{N^{\varphi}}{(1-\alpha)\, N^{-\alpha}}\frac{1}{2}\alpha(1-\alpha)\, N^{-\alpha-1}\right)\hat{N}_t^2 \\
&+ \frac{N^{\varphi}N^{\alpha}}{(1-\alpha)\, N^{-\alpha}}\hat{A}_t + N^{\varphi}\hat{N}_t\hat{A}_t - \frac{N^{\varphi}}{(1-\alpha)\, N^{-\alpha}}(1-\alpha)C\hat{S}_t^w\,.
\end{aligned}
\tag{D.15}
$$

Next, consider the law of motion for the wage dispersion term S_t^w:

$$
\begin{aligned}
S_t^W &= \int_0^1 \left(\frac{W_t^j}{W_t}\right)^{-\varepsilon_w} dj = (1-\theta)\left(\frac{W_t^*}{W_t}\right)^{-\varepsilon_w} + \theta \int_0^1 \left(\frac{W_{t-1}^j}{W_t}\right)^{-\varepsilon_w} dj \\
&= (1-\theta)\left(\frac{W_t^*}{W_t}\right)^{-\varepsilon_w} + \theta \left(\frac{W_{t-1}}{W_t}\right)^{-\varepsilon_w} \int_0^1 \left(\frac{W_{t-1}^j}{W_{t-1}}\right)^{-\varepsilon_w} dj \\
&= (1-\theta)\left(\Pi_t^{w*}\right)^{-\varepsilon_w} + \theta \left(\frac{W_{t-1}}{W_t}\right)^{-\varepsilon_w} S_{t-1}^W \\
&= (1-\theta)\left(\Pi_t^{w*}\right)^{-\varepsilon_w} + \theta \left(\Pi_t^w\right)^{\varepsilon_w} S_{t-1}^W \,.
\end{aligned} \tag{D.16}
$$

A second order Taylor approximation yields

$$
\begin{aligned}
\hat{S}_t^W =& (1-\theta_w)(-\varepsilon_w)\left(\Pi_t^{w*}\right)^{-\varepsilon_w - 1}\left(\Pi_t^{w*} - \Pi^{w*}\right) + \varepsilon_w \theta_w (\Pi^w)^{\varepsilon_w - 1} S^W \left(\Pi_t^w - \Pi^w\right) + \theta_w (\Pi_t^w)^{\varepsilon_w}\left(S_t^W - S^w\right) \\
&+ \frac{1}{2}\begin{bmatrix} (1-\theta_w)(-\varepsilon_w)(-\varepsilon_w - 1)\left(\Pi_t^{w*}\right)^{-\varepsilon_w - 2}(\Pi_t^{w*} - \Pi^{w*})^2 \\ +\varepsilon_w(\varepsilon_w - 1)\theta_w (\Pi_t^w)^{\varepsilon_w - 2} S^W (\Pi_t^w - \Pi^w)^2 \end{bmatrix} \\
&+ \varepsilon_w \theta_w (\Pi^w)^{\varepsilon_w - 1}\left(\Pi_t^w - \Pi^w\right)\left(S_{t-1}^W - S^w\right) \\
=& (1-\theta_w)(-\varepsilon_w)\left(\Pi_t^{w*} - \Pi^{w*}\right) + \varepsilon_w \theta_w \left(\Pi_t^w - \Pi^w\right) + \theta_w \left(S_{t-1}^W - S^w\right) \\
&+ \frac{1}{2}\left[(1-\theta_w)\varepsilon_w(\varepsilon_w + 1)\left(\Pi_t^{w*} - \Pi^{w*}\right)^2 + \varepsilon_w(\varepsilon_w - 1)\theta_w (\Pi_t^w - \Pi^w)^2\right] \\
&+ \varepsilon_w \theta_w \left(\Pi_t^w - \Pi^w\right)\left(S_{t-1}^W - S^w\right) \\
=& \varepsilon_w \left(\theta_w \left(\Pi_t^w - \Pi^w\right) - (1-\theta_w)\left(\Pi_t^{w*} - \Pi^{w*}\right)\right) + \theta_w \left(S_{t-1}^W - S^w\right) \\
&+ \frac{1}{2}\left[(1-\theta_w)\varepsilon_w(\varepsilon_w + 1)\left(\Pi_t^{w*} - \Pi^{w*}\right)^2 + \varepsilon_w(\varepsilon_w - 1)\theta_w (\Pi_t^w - \Pi^w)^2\right] \\
&+ \varepsilon_w \theta_w \left(\Pi_t^w - \Pi^w\right)\left(S_{t-1}^W - S^w\right) \,,
\end{aligned} \tag{D.17}
$$

where we again imposed a zero inflation steady state. The evolution of wages is given by

$$
W_t = \left[(1-\theta_w)\left(W_t^*\right)^{1-\varepsilon_w} + \theta_w W_{t-1}^{1-\varepsilon_w}\right]^{\frac{1}{1-\varepsilon_w}} \tag{D.18}
$$

so that

$$
1 = (1-\theta_w)\left(\Pi_t^{w*}\right)^{1-\varepsilon_w} + \theta_w (\Pi_t^w)^{\varepsilon_w - 1} \,. \tag{D.19}
$$

A first order approximation yields

$$
0 = (1-\theta_w)(1-\varepsilon_w)\left(\Pi_t^{w*}\right)^{-\varepsilon_w}\left(\Pi_t^{w*} - \Pi^{w*}\right) + \theta_w(\varepsilon_w - 1)\left(\Pi^w\right)^{\varepsilon_w - 2}\left(\Pi_t^w - \Pi^w\right) \tag{D.20}
$$

so that

$$
\theta_w \left(\Pi_t^w - \Pi^w\right) = (1-\theta_w)\left(\Pi_t^{w*} - \Pi^{w*}\right) \,, \tag{D.21}
$$

which implies

$$\left(\frac{\theta_w}{(1-\theta_w)}\right)^2 (\Pi_t^w - \Pi^w)^2 = (\Pi_t^{w*} - \Pi^{w*})^2 \tag{D.22}$$

A second-order approximation of (D.19) yields

$$\begin{aligned}
0 =& (1-\theta_w)(1-\varepsilon_w)(\Pi_t^{w*})^{-\varepsilon_w}(\Pi_t^{w*}-\Pi^{w*}) + \theta_w(\varepsilon_w - 1)(\Pi^w)^{\varepsilon_w - 2}(\Pi_t^w - \Pi^w) \\
&+ \frac{1}{2}\begin{bmatrix} (1-\theta_w)(1-\varepsilon_w)(-\varepsilon_w)(\Pi_t^{w*})^{-\varepsilon_w - 1}(\Pi_t^{w*}-\Pi^{w*})^2 \\ +\theta_w(\varepsilon_w-1)(\varepsilon_w-2)(\Pi^w)^{\varepsilon_w-3}(\Pi_t^w-\Pi^w)^2 \end{bmatrix} \\
=& (1-\theta_w)(\Pi_t^{w*}-\Pi^{w*}) - \theta_w(\Pi_t^w - \Pi^w) \\
&+ \frac{1}{2}\left[(1-\theta_w)(-\varepsilon_w)(\Pi_t^{w*}-\Pi^{w*})^2 - \theta_w(\varepsilon_w-2)(\Pi_t^w-\Pi^w)^2\right]
\end{aligned} \tag{D.23}$$

so that

$$\begin{aligned}
&\theta_w(\Pi_t^w-\Pi^w) - (1-\theta_w)(\Pi_t^{w*}-\Pi^{w*}) \\
&= \frac{1}{2}\left[(1-\theta_w)(-\varepsilon_w)(\Pi_t^{w*}-\Pi^{w*})^2 - \theta_w(\varepsilon_w-2)(\Pi_t^w-\Pi^w)^2\right] .
\end{aligned} \tag{D.24}$$

Inserting into (D.17), we get:

$$\begin{aligned}
\hat{S}_t^W =& \frac{1}{2}\varepsilon_w\left[(1-\theta_w)(-\varepsilon_w)(\Pi_t^{w*}-\Pi^{w*})^2 - \theta_w(\varepsilon_w-2)(\Pi_t^w-\Pi^w)^2\right] + \theta_w\left(S_{t-1}^W - S^w\right) \\
&+ \frac{1}{2}\left[(1-\theta_w)\varepsilon_w(\varepsilon_w+1)(\Pi_t^{w*}-\Pi^{w*})^2 + \varepsilon_w(\varepsilon_w-1)\theta_w(\Pi_t^w-\Pi^w)^2\right] \\
&+ \varepsilon_w\theta_w(\Pi_t^w-\Pi^w)\left(S_{t-1}^W - S^w\right) .
\end{aligned} \tag{D.25}$$

This shows the well-known result that $\hat{S}_t^W$ is 0 up to first order in a zero inflation steady state. Thus, up to second order, we can drop the last term as it is of third order. Now we can use (D.22) to get

$$
\begin{aligned}
\hat{S}_t^W =&\theta_w \left(S_{t-1}^W - S^w\right) + \frac{1}{2}\varepsilon_w \left[(1-\theta_w)\,(-\varepsilon_w)\left(\frac{\theta_w}{(1-\theta_w)}\right)^2 (\Pi_t^w - \Pi^w)^2 - \theta_w\,(\varepsilon_w - 2)\,(\Pi_t^w - \Pi^w)^2\right] \\
&+ \frac{1}{2}\left[(1-\theta_w)\,\varepsilon_w\,(\varepsilon_w+1)\left(\frac{\theta_w}{(1-\theta_w)}\right)^2 (\Pi_t^w - \Pi^w)^2 + \varepsilon_w\,(\varepsilon_w - 1)\,\theta_w (\Pi_t^w - \Pi^w)^2\right] \\
=&\theta_w \left(S_{t-1}^W - S^w\right) + \frac{1}{2}\begin{bmatrix} (1-\theta_w)\left(\dfrac{\theta_w}{(1-\theta_w)}\right)^2 (\Pi_t^w - \Pi^w)^2 \left(-\varepsilon_w^2 + \varepsilon_w^2 + \varepsilon_w\right) \\ -\theta_w (\Pi_t^w - \Pi^w)^2 \left(-\varepsilon_w^2 - 2\varepsilon_w + \varepsilon_w^2 - \varepsilon_w\right) \end{bmatrix} \\
=&\theta_w \left(S_{t-1}^W - S^w\right) + \frac{1}{2}\left[\frac{\theta_w^2}{(1-\theta_w)}(\Pi_t^w - \Pi^w)^2 \varepsilon_w + \theta_w (\Pi_t^w - \Pi^w)^2 \varepsilon_w\right] \\
=&\theta_w \left(S_{t-1}^W - S^w\right) + \frac{1}{2}\varepsilon_w \theta_w \left[\frac{\theta_w}{(1-\theta_w)} + 1\right] (\Pi_t^w - \Pi^w)^2 \\
=&\theta_w \left(S_{t-1}^W - S^w\right) + \frac{1}{2}\varepsilon_w \theta_w \left[\frac{\theta_w}{(1-\theta_w)} + \frac{(1-\theta_w)}{(1-\theta_w)}\right] (\Pi_t^w - \Pi^w)^2 \\
=&\theta_w \left(S_{t-1}^W - S^w\right) + \frac{1}{2}\frac{\varepsilon_w \theta_w}{1-\theta_w}(\Pi_t^w - \Pi^w)^2 \\
=&\theta_w \hat{S}_{t-1}^W + \frac{1}{2}\frac{\varepsilon_w \theta_w}{1-\theta_w}\left(\hat{\Pi}_t^w\right)^2 .
\end{aligned}
\tag{D.26}
$$

Iterating this equation forward from time t_0 onwards yields

$$
\begin{aligned}
\hat{S}_{t_0}^W =&\theta_w \hat{S}_{t_0-1}^W + \frac{1}{2}\frac{\varepsilon_w \theta_w}{1-\theta_w}\left(\hat{\Pi}_{t_0}^w\right)^2 \\
\hat{S}_{t_0+1}^W =&\theta_w \left(\theta_w \hat{S}_{t_0-1}^W + \frac{1}{2}\frac{\varepsilon_w \theta_w}{1-\theta_w}\left(\hat{\Pi}_{t_0}^w\right)^2\right) + \frac{1}{2}\frac{\varepsilon_w \theta_w}{1-\theta_w}\left(\hat{\Pi}_{t_0+1}^w\right)^2 \\
\hat{S}_{t_0+2}^W =& \left(\theta_w \left(\theta_w \hat{S}_{t_0-1}^W + \frac{1}{2}\frac{\varepsilon_w \theta_w}{1-\theta_w}\left(\hat{\Pi}_{t_0}^w\right)^2\right) + \frac{1}{2}\frac{\varepsilon_w \theta_w}{1-\theta_w}\left(\hat{\Pi}_{t_0+1}^w\right)^2\right) + \frac{1}{2}\frac{\varepsilon_w \theta_w}{1-\theta_w}\left(\hat{\Pi}_{t_0+2}^w\right)^2
\end{aligned}
\tag{D.27}
$$

so that the discounted sum is given by

$$
\begin{aligned}
&\sum_{t=t_0}^{\infty} \beta^{t-t_0} \hat{S}_t^W \\
&= \sum_{t=t_0}^{\infty} (\beta\theta_w)^{t-t_0} \left(\frac{1}{2} \frac{\varepsilon_w \theta_w}{1-\theta_w} \left(\hat{\Pi}_{t_0}^w \right)^2 + \theta_w \hat{S}_{t_0-1}^W \right) + \sum_{t=t_0+1}^{\infty} \beta^{t-t_0} \theta_w^{t-t_0+1} \left(\frac{1}{2} \frac{\varepsilon_w \theta_w}{1-\theta_w} \left(\hat{\Pi}_{t_0+1}^w \right)^2 \right) + \ldots \\
&= \sum_{t=t_0}^{\infty} (\beta\theta_w)^{t-t_0} \theta_w \hat{S}_{t_0-1}^W + \left(\frac{1}{2} \frac{\varepsilon_w \theta_w}{1-\theta_w} \left(\hat{\Pi}_{t_0}^w \right)^2 \right) \sum_{t=t_0}^{\infty} (\beta\theta_w)^{t-t_0} + \left(\frac{1}{2} \frac{\varepsilon_w \theta_w}{1-\theta_w} \left(\hat{\Pi}_{t_0+1}^w \right)^2 \right) \sum_{t=t_0+1}^{\infty} (\beta\theta_w)^{t-t_0} \theta_w + \ldots \\
&= \frac{\theta_w}{1-\beta\theta_w} \hat{S}_{t_0-1}^W + \left(\frac{1}{2} \frac{\varepsilon_w \theta_w}{1-\theta_w} \left(\hat{\Pi}_{t_0}^w \right)^2 \right) \frac{1}{1-\beta\theta_w} + \left(\frac{1}{2} \frac{\varepsilon_w \theta_w}{1-\theta_w} \left(\hat{\Pi}_{t_0+1}^w \right)^2 \right) \frac{\beta}{1-\beta\theta_w} + \ldots \\
&= \frac{\theta_w}{1-\beta\theta_w} \hat{S}_{t_0-1}^W + \frac{1}{1-\beta\theta_w} \left[\left(\frac{1}{2} \frac{\varepsilon_w \theta_w}{1-\theta_w} \left(\hat{\Pi}_{t_0}^w \right)^2 \right) + \left(\frac{1}{2} \frac{\varepsilon_w \theta_w}{1-\theta_w} \left(\hat{\Pi}_{t_0+1}^w \right)^2 \right) \beta + \ldots \right] \\
&= \frac{\theta_w}{1-\beta\theta_w} \hat{S}_{t_0-1}^W + \frac{1}{1-\beta\theta_w} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left(\frac{1}{2} \frac{\varepsilon_w \theta_w}{1-\theta_w} \left(\hat{\Pi}_t^w \right)^2 \right) \\
&= \frac{\theta_w}{1-\beta\theta_w} \hat{S}_{t_0-1}^W + \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{1}{2} \left(\frac{\varepsilon_w \theta_w}{(1-\beta\theta_w)(1-\theta_w)} \left(\hat{\Pi}_t^w \right)^2 \right) .
\end{aligned}
\tag{D.28}
$$

D.2 Comparison

Comparing welfare under Calvo (D.15) and Rotemberg (D.8) shows the difference is given by

$$
\begin{aligned}
\Delta U^{SGU} &= \hat{U}_{Calvo}^{SGU} - \hat{U}_{Rotemberg}^{SGU} \\
&= \frac{N^{\varphi}}{(1-\alpha) N^{-\alpha}} C \frac{\phi_w^{SGU}}{2} \left(\hat{\Pi}_t^w \right)^2 - \frac{N^{\varphi}}{(1-\alpha) N^{-\alpha}} (1-\alpha) C \hat{S}_t^w \\
&= \frac{N^{\varphi}}{(1-\alpha) N^{-\alpha}} C \left(\frac{\phi_w^{SGU}}{2} \left(\hat{\Pi}_t^w \right)^2 - (1-\alpha) \hat{S}_t^w \right) .
\end{aligned}
\tag{D.29}
$$

Using (D.28) and

$$
\frac{N^{\varphi} C}{(1-\alpha) N^{-\alpha}} = \frac{N^{\varphi} N^{1-\alpha}}{(1-\alpha) N^{-\alpha}} = \frac{N^{1+\varphi}}{(1-\alpha)} , \tag{D.30}
$$

we can write the welfare difference as

$$
\begin{aligned}
&\sum_{t=t_0}^{\infty} \beta^{t-t_0} \Delta U^{SGU} \\
&= \frac{N^{1+\varphi}}{(1-\alpha)} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left(\frac{\phi_w^{SGU}}{2} \left(\hat{\Pi}_t^w\right)^2 - (1-\alpha)\,\hat{S}_t^w \right) \\
&= \frac{N^{1+\varphi}}{(1-\alpha)} \left[\begin{array}{l} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{\phi_w^{SGU}}{2} \left(\hat{\Pi}_t^w\right)^2 - (1-\alpha)\, \frac{\theta_w}{1-\beta\theta_w} \hat{S}_{t_0-1}^W \\ -(1-\alpha) \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{1}{2} \left(\frac{\varepsilon_w \theta_w}{(1-\beta\theta_w)(1-\theta_w)} \left(\hat{\Pi}_t^w\right)^2 \right) \end{array} \right] \\
&= \frac{N^{1+\varphi}}{(1-\alpha)} \left[-(1-\alpha)\, \frac{\theta_w}{1-\beta\theta_w} \hat{S}_{t_0-1}^W + \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{1}{2} \left(\phi_w^{SGU} - (1-\alpha)\, \frac{\varepsilon_w \theta_w}{(1-\beta\theta_w)(1-\theta_w)} \right) \left(\hat{\Pi}_t^w\right)^2 \right].
\end{aligned}
\tag{D.31}
$$

Assuming that initial price dispersion is 0, i.e. $\hat{S}_{t_0-1}^W = 0$, this simplifies to

$$
\begin{aligned}
\sum_{t=t_0}^{\infty} \beta^{t-t_0} \Delta U^{SGU} =& \frac{N^{1+\varphi}}{(1-\alpha)} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{1}{2} \left(\phi_w^{SGU} - (1-\alpha)\, \frac{\varepsilon_w \theta_w}{(1-\beta\theta_w)(1-\theta_w)} \right) \left(\hat{\Pi}_t^w\right)^2 \\
=& \frac{N^{1+\varphi}}{(1-\alpha)} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{1}{2} \left(\frac{(\varepsilon_w - 1)(1-\tau^n)\aleph}{(1-\theta_w)(1-\beta\theta_w)} \theta_w - (1-\alpha)\, \frac{\varepsilon_w \theta_w}{(1-\beta\theta_w)(1-\theta_w)} \right) \left(\hat{\Pi}_t^w\right)^2 \\
=& N^{1+\varphi} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{1}{2} \left(\frac{(\varepsilon_w - 1)(1-\tau^n)\aleph}{(1-\alpha)(1-\theta_w)(1-\beta\theta_w)} \theta_w - \frac{\varepsilon_w \theta_w}{(1-\beta\theta_w)(1-\theta_w)} \right) \left(\hat{\Pi}_t^w\right)^2,
\end{aligned}
\tag{D.32}
$$

where the second line uses that the slope of the wage Phillips Curve is identical with

$$
\phi_w^{SGU} = \frac{(\varepsilon_w - 1)(1-\tau^n)\aleph}{(1-\theta_w)(1-\beta\theta_w)} \theta_w \tag{D.33}
$$

With $\aleph = (1-\alpha)$ and $(1-\tau^n) = \frac{\varepsilon_w}{(\varepsilon_w - 1)}$, i.e. if the monopolistic distortion is counteracted by appropriate subsidies:

$$
\sum_{t=t_0}^{\infty} \beta^{t-t_0} \Delta U^{SGU} = N^{1+\varphi} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{1}{2} \left(\frac{\varepsilon_w \theta_w}{(1-\theta_w)(1-\beta\theta_w)} - \frac{\varepsilon_w \theta_w}{(1-\beta\theta_w)(1-\theta_w)} \right) \left(\hat{\Pi}_t^w\right)^2 = 0. \tag{D.34}
$$

Thus, in the case of an undistorted steady state, i.e. when the labor tax undoes the effect of monopolistic competition, the welfare losses conditional on initial wage dispersion being 0 are identical between the Rotemberg and Calvo frameworks. This completes the proof of Proposition 2 for the SGU framework.

Finally, consider unconditional welfare. Taking the unconditional expectations of equation (D.26) we get

$$E\hat{S}_t^W = E\theta_w \hat{S}_{t-1}^W + \frac{1}{2}\frac{\varepsilon_w \theta_w}{1-\theta_w} E\left(\hat{\Pi}_t^w\right)^2 \tag{D.35}$$

so that the unconditional mean of the price dispersion term is given by:

$$E\hat{S}_t^W = \frac{1}{2}\frac{\varepsilon_w \theta_w}{\left(1-\theta_w\right)^2} E\left(\hat{\Pi}_t^w\right)^2 . \tag{D.36}$$

Hence, the average wage dispersion in the stochastic model is not 0. Taking the unconditional expectations in (D.31) and using the previous result, we obtain:

$$\begin{aligned}
E\sum_{t=t_0}^{\infty}\beta^{t-t_0}\Delta U^{SGU} =& \frac{N^{1+\varphi}}{(1-\alpha)}\begin{bmatrix} -\left(1-\alpha\right)\dfrac{\theta_w}{1-\beta\theta_w}E\hat{S}_{t_0-1}^W \\ +\displaystyle\sum_{t=t_0}^{\infty}\beta^{t-t_0}\frac{1}{2}\left(\phi_w^{SGU} - \left(1-\alpha\right)\frac{\varepsilon_w\theta_w}{\left(1-\beta\theta_w\right)\left(1-\theta_w\right)}\right)E\left(\hat{\Pi}_t^w\right)^2 \end{bmatrix} \\
=& \frac{N^{1+\varphi}}{(1-\alpha)}\begin{bmatrix} -\left(1-\alpha\right)\dfrac{\theta_w}{1-\beta\theta_w}\dfrac{1}{2}\dfrac{\varepsilon_w\theta_w}{\left(1-\theta_w\right)^2}E\left(\hat{\Pi}_t^w\right)^2 \\ +\dfrac{1}{2}\left(-\left(1-\alpha\right)\dfrac{\varepsilon_w\theta_w}{\left(1-\beta\theta_w\right)\left(1-\theta_w\right)}\right)\dfrac{1}{1-\beta}E\left(\hat{\Pi}_t^w\right)^2 \end{bmatrix} \\
=& \frac{1}{2}\frac{N^{1+\varphi}}{(1-\alpha)}\begin{pmatrix} \phi_w^{SGU} - \left(1-\alpha\right)\dfrac{\varepsilon_w\theta_w}{\left(1-\beta\theta_w\right)\left(1-\theta_w\right)} \\ -\left(1-\beta\right)\left(1-\alpha\right)\dfrac{\theta_w}{1-\beta\theta_w}\dfrac{\varepsilon_w\theta_w}{\left(1-\theta_w\right)^2} \end{pmatrix}\frac{1}{1-\beta}E\left(\hat{\Pi}_t^w\right)^2 .
\end{aligned} \tag{D.37}$$

Again imposing identical wage PC slopes via (D.33), we get

$$E\sum_{t=t_0}^{\infty}\beta^{t-t_0}\Delta U^{SGU} = \frac{1}{2}\frac{N^{1+\varphi}}{(1-\alpha)}\phi_w^{SGU}\begin{pmatrix} 1-\dfrac{\left(1-\alpha\right)\varepsilon_w}{\left(\varepsilon_w-1\right)\aleph\left(1-\tau^n\right)} \\ -\dfrac{\left(1-\beta\right)\left(1-\alpha\right)\theta_w}{\left(1-\theta_w\right)}\dfrac{\left(1-\alpha\right)\varepsilon_w}{\left(\varepsilon_w-1\right)\aleph\left(1-\tau^n\right)} \end{pmatrix}\frac{1}{1-\beta}E\left(\hat{\Pi}_t^w\right)^2. \tag{D.38}$$

With an undistorted steady state characterized by $\aleph = (1-\alpha)$ and $(1-\tau^n) = \frac{\varepsilon_w}{(\varepsilon_w - 1)}$:

$$
\begin{aligned}
&E\sum_{t=t_0}^{\infty}\beta^{t-t_0}\Delta U^{SGU}\\
&=\frac{1}{2}\frac{N^{1+\varphi}}{(1-\alpha)}\phi_w^{SGU}\left(1-\frac{(1-\alpha)\,\varepsilon_w}{(\varepsilon_w-1)\,\aleph\,(1-\tau^n)}-\frac{(1-\beta)\,(1-\alpha)\,\theta_w}{(1-\theta_w)}\frac{(1-\alpha)\,\varepsilon_w}{(\varepsilon_w-1)\,\aleph\,(1-\tau^n)}\right)\frac{1}{1-\beta}E\Big(\hat{\Pi}_t^w\Big)^2\\
&=\frac{1}{2}\frac{N^{1+\varphi}}{(1-\alpha)}\phi_w^{SGU}\left(-\frac{(1-\beta)\,(1-\alpha)\,\theta_w}{(1-\theta_w)}\right)\frac{1}{1-\beta}E\Big(\hat{\Pi}_t^w\Big)^2\\
&=\frac{1}{2}N^{1+\varphi}\phi_w^{SGU}\left(-\frac{(1-\beta)\,\theta_w}{(1-\theta_w)}\right)\frac{1}{1-\beta}E\Big(\hat{\Pi}_t^w\Big)^2 .
\end{aligned}
\tag{D.39}
$$

Noting that

$$
1-\frac{1-\beta\theta_w}{1-\theta_w}=\frac{1-\theta_w}{1-\theta_w}-\frac{1-\beta\theta_w}{1-\theta_w}=\frac{-\theta_w+\beta\theta_w}{1-\theta_w}=\frac{-\theta_w\,(1-\beta)}{1-\theta_w}\,, \tag{D.40}
$$

we get that

$$
E\sum_{t=t_0}^{\infty}\beta^{t-t_0}\Delta U^{SGU}=\frac{1}{2}N^{1+\varphi}\phi_w^{SGU}\left(1-\frac{1-\beta\theta_w}{1-\theta_w}\right)\frac{1}{1-\beta}E\Big(\hat{\Pi}_t^w\Big)^2<0\,. \tag{D.41}
$$

Hence, Calvo wage setting is associated with higher unconditional welfare losses. This completes the proof of Proposition 3for the SGU case.

D.3 EHL

In the EHL case, workers supply differentiated goods, which complicates aggregation in the Calvo case. In the symmetric Rotemberg equilibrium, the aggregate felicity function is still

$$
U_{Rotemberg}^{EHL}=\int_0^1 U_t\,(j)\,dj=\frac{C_t^{1-\sigma}-1}{1-\sigma}-\frac{N_t^{1+\varphi}}{1+\varphi}=U_{Rotemberg}^{SGU} \tag{D.42}
$$

Aggregation in the Calvo case is more involved. We obtain

$$\begin{aligned}
U^{EHL}_{Calvo} &= \int_0^1 \frac{\left(C_t^j\right)^{1-\sigma}-1}{1-\sigma} - \frac{\left(N_t^j\right)^{1+\varphi}}{1+\varphi} dj \\
&= \frac{C_t^{1-\sigma}-1}{1-\sigma} - \int_0^1 \left(\frac{\left(N_t^j\right)^{1+\varphi}}{1+\varphi}\right) dj \\
&= \frac{C_t^{1-\sigma}-1}{1-\sigma} - \int_0^1 \left(\frac{\left(\left(\frac{W_t^j}{W_t}\right)^{-\varepsilon} N_t^d\right)^{1+\varphi}}{1+\varphi}\right) dj \\
&= \frac{C_t^{1-\sigma}-1}{1-\sigma} - \frac{\left(N_t^d\right)^{1+\varphi}}{1+\varphi} \int_0^1 \left(\left(\frac{W_t^j}{W_t}\right)^{-\varepsilon}\right)^{1+\varphi} dj \\
&= \frac{C_t^{1-\sigma}-1}{1-\sigma} - \frac{\left(N_t^d\right)^{1+\varphi}}{1+\varphi} \int_0^1 \left(\frac{W_t^j}{W_t}\right)^{-\varepsilon(1+\varphi)} dj \\
&= \left(\frac{C_t^{1-\sigma}-1}{1-\sigma} - \frac{\left(N_t^d\right)^{1+\varphi}}{1+\varphi} X_t^W\right) \\
&= \frac{C_t^{1-\sigma}-1}{1-\sigma} - \frac{\left(\frac{N_t}{S_t^W}\right)^{1+\varphi}}{1+\varphi} X_t^W \,, \qquad \text{(D.43)}
\end{aligned}$$

where the second equality uses the complete markets assumption and where the auxiliary variable X_t^W has the recursive representation

$$\begin{aligned}
X_t^W \equiv \int_0^1 \left(\frac{W_t^j}{W_t}\right)^{-\varepsilon_w(1+\varphi)} dj &= (1-\theta)\left(\frac{W_t^*}{W_t}\right)^{-\varepsilon_w(1+\varphi)} + \theta \int_0^1 \left(\frac{W_{t-1}^j}{W_t}\right)^{-\varepsilon_w(1+\varphi)} dj \\
&= (1-\theta)\left(\frac{W_t^*}{W_t}\right)^{-\varepsilon_w(1+\varphi)} + \theta\left(\frac{W_{t-1}}{W_t}\right)^{-\varepsilon_w} \int_0^1 \left(\frac{W_{t-1}^j}{W_{t-1}}\right)^{-\varepsilon_w(1+\varphi)} dj \\
&= (1-\theta)\left(\Pi_t^{w*}\right)^{-\varepsilon_w(1+\varphi)} + \theta\left(\frac{W_{t-1}}{W_t}\right)^{-\varepsilon_w(1+\varphi)} X_{t-1}^W \\
&= (1-\theta)\left(\Pi_t^{w*}\right)^{-\varepsilon_w(1+\varphi)} + \theta(\Pi_t^w)^{\varepsilon_w(1+\varphi)} X_{t-1}^W \,, \qquad \text{(D.44)}
\end{aligned}$$

Thus, compared to the Rotemberg case in (D.42), the disutility of labor term is different. A second-order approximation to the disutility of labor term yields

$$
\begin{aligned}
\frac{\left(\frac{N_t}{S_t^W}\right)^{1+\varphi}}{1+\varphi} X_t^W \approx & \frac{N^{1+\varphi}}{1+\varphi} + \frac{N^{\varphi}}{\left(S^W\right)^{1+\varphi}} X^W \left(N_t - N\right) - N^{1+\varphi} \left(S^W\right)^{-(1+\varphi)-1} X^W \left(S_t^W - S^W\right) \\
& + \frac{1}{2} \frac{X^W}{\left(S^W\right)^{1+\varphi}} N^{\varphi-1} \varphi (N_t - N)^2 + \frac{\left(\frac{N}{S^W}\right)^{1+\varphi}}{1+\varphi} \left(X_t^W - X^W\right) \\
= & \frac{N^{1+\varphi}}{1+\varphi} + N^{\varphi} \hat{N}_t - N^{1+\varphi} \hat{S}_t^W + \frac{1}{2} N^{\varphi-1} \varphi \hat{N}_t^2 + \frac{N^{1+\varphi}}{1+\varphi} \hat{X}_t^W ,
\end{aligned} \tag{D.45}
$$

where we used that the dispersion terms $\hat{S}_t^w$ and $\hat{X}_t^w$ are 0 up to first order, as we will show below. As the resource constraint and the production function are the same as in the SGU case, we get

$$
\begin{aligned}
\hat{U}_{Calvo}^{EHL} = & C^{-\sigma} \hat{C}_t - \frac{1}{2} C^{-\sigma-1} \sigma \hat{C}_t^2 - N^{\varphi} \hat{N}_t + N^{1+\varphi} \left(S_t^W - S^W\right) - \frac{1}{2} N^{\varphi-1} \varphi \hat{N}_t^2 - \frac{N^{1+\varphi}}{1+\varphi} \left(X_t^W - X^W\right) \\
= & C^{-\sigma} \hat{C}_t - \frac{1}{2} C^{-\sigma-1} \sigma \hat{C}_t^2 - N^{\varphi} \left(\frac{1}{(1-\alpha) N^{-\alpha}} \begin{bmatrix} \hat{C}_t + (1-\alpha) C \hat{S}_t^w - N^{1-\alpha} \hat{A}_t - (1-\alpha) N^{-\alpha} \hat{N}_t \\ - (1-\alpha) N^{-\alpha} \hat{N}_t \hat{A}_t + \frac{1}{2} \alpha (1-\alpha) N^{-\alpha-1} \hat{N}_t^2 \end{bmatrix} \right) \\
& + N^{1+\varphi} \left(S_t^W - S^W\right) - \frac{1}{2} N^{\varphi-1} \varphi \hat{N}_t^2 - \frac{N^{1+\varphi}}{1+\varphi} \left(X_t^W - X^W\right) \\
= & \left(C^{-\sigma} - \frac{N^{\varphi}}{(1-\alpha) N^{-\alpha}} \right) \hat{C}_t - \frac{1}{2} C^{-\sigma-1} \sigma \hat{C}_t^2 - \left(\frac{1}{2} N^{\varphi-1} \varphi - \frac{N^{\varphi}}{(1-\alpha) N^{-\alpha}} \frac{1}{2} \alpha (1-\alpha) N^{-\alpha-1} \right) \hat{N}_t^2 \\
& + \frac{N^{\varphi} N^{\alpha}}{(1-\alpha) N^{-\alpha}} \hat{A}_t + N^{\varphi} \hat{N}_t \hat{A}_t - \left(\frac{N^{\varphi}}{N^{-\alpha}} C - N^{1+\varphi} \right) \hat{S}_t^w - \frac{N^{1+\varphi}}{1+\varphi} \hat{X}_t^W .
\end{aligned} \tag{D.46}
$$

Thus, the period utility difference between Calvo and Rotemberg is given by

$$
\begin{aligned}
\hat{U}_{Calvo}^{EHL} - \hat{U}_{Rotemberg}^{EHL} = & \frac{N^{\varphi}}{(1-\alpha) N^{-\alpha}} C \frac{\phi_w^{EHL}}{2} \left(\hat{\Pi}_t^w\right)^2 + \left(\frac{N^{\varphi}}{N^{-\alpha}} C - N^{1+\varphi} \right) \hat{S}_t^w - \frac{N^{1+\varphi}}{1+\varphi} \hat{X}_t^W \\
= & \frac{N^{\varphi}}{(1-\alpha) N^{-\alpha}} C \left(\frac{\phi_w^{EHL}}{2} \left(\hat{\Pi}_t^w\right)^2 - (1-\alpha) \hat{S}_t^w \right) + N^{1+\varphi} \left(\hat{S}_t^w - \frac{1}{1+\varphi} \hat{X}_t^W \right) .
\end{aligned} \tag{D.47}
$$

As in steady state

$$
\frac{N^{\varphi} C}{(1-\alpha) N^{-\alpha}} = \frac{N^{\varphi} N^{1-\alpha}}{(1-\alpha) N^{-\alpha}} = \frac{N^{1+\varphi}}{(1-\alpha)} \tag{D.48}
$$

we can simplify the welfare loss to

$$
\Delta U^{EHL} = \hat{U}_{Calvo}^{EHL} - \hat{U}_{Rotemberg}^{EHL} = \frac{N^{1+\varphi}}{(1-\alpha)} \left(\frac{\phi_w^{EHL}}{2} \left(\hat{\Pi}_t^w\right)^2 - \frac{(1-\alpha)}{1+\varphi} \hat{X}_t^W \right) . \tag{D.49}
$$

The derivation of the second-order approximation to the auxiliary variable

$$X_t^W = (1-\theta_W)\left(\Pi_t^{w*}\right)^{-\varepsilon_w(1+\varphi)} + \theta_W(\Pi_t^w)^{\varepsilon_w(1+\varphi)} X_{t-1}^W \tag{D.50}$$

follows the lines of the one for S_t^w. A second-order approximation yields:

$$\begin{aligned}
\hat{X}_t^W =& (1-\theta_w)\left(-\varepsilon_w(1+\varphi)\right)\left(\Pi_t^{w*}\right)^{-\varepsilon_w(1+\varphi)-1}\left(\Pi_t^{w*}-\Pi^{w*}\right) + \varepsilon_w(1+\varphi)\,\theta_w(\Pi^w)^{\varepsilon_w(1+\varphi)-1}X^W\left(\Pi_t^w-\Pi^w\right)\\
&+ \theta_w(\Pi_t^w)^{\varepsilon_w(1+\varphi)}\left(X_{t-1}^W - X^w\right)\\
&+ \frac{1}{2}\begin{bmatrix}(1-\theta_w)\left(-\varepsilon_w(1+\varphi)\right)\left(-\varepsilon_w(1+\varphi)-1\right)\left(\Pi_t^{w*}\right)^{-\varepsilon_w(1+\varphi)-2}(\Pi_t^{w*}-\Pi^{w*})^2\\ +\varepsilon_w(1+\varphi)\left(\varepsilon_w(1+\varphi)-1\right)\theta_w(\Pi_t^w)^{\varepsilon_w(1+\varphi)-2}X^W(\Pi_t^w-\Pi^w)^2\end{bmatrix}\\
&+ \varepsilon_w(1+\varphi)\,\theta_w(\Pi^w)^{\varepsilon_w(1+\varphi)-1}\left(\Pi_t^w-\Pi^w\right)\left(X_{t-1}^W - X^w\right)\\
=& (1-\theta_w)\left(-\varepsilon_w(1+\varphi)\right)\left(\Pi_t^{w*}-\Pi^{w*}\right) + \varepsilon_w(1+\varphi)\,\theta_w\left(\Pi_t^w-\Pi^w\right) + \theta_w\left(X_{t-1}^W - X^w\right)\\
&+ \frac{1}{2}\begin{bmatrix}(1-\theta_w)\,\varepsilon_w(1+\varphi)\left(\varepsilon_w(1+\varphi)+1\right)\left(\Pi_t^{w*}-\Pi^{w*}\right)^2\\ +\varepsilon_w(1+\varphi)\left(\varepsilon_w(1+\varphi)-1\right)\theta_w(\Pi_t^w-\Pi^w)^2\end{bmatrix}\\
&+ \varepsilon_w(1+\varphi)\,\theta_w\left(\Pi_t^w-\Pi^w\right)\left(X_{t-1}^W - X^w\right)\\
=& \varepsilon_w(1+\varphi)\left(\theta_w\left(\Pi_t^w-\Pi^w\right) - (1-\theta_w)\left(\Pi_t^{w*}-\Pi^{w*}\right)\right) + \theta_w\left(X_{t-1}^W - X^w\right)\\
&+ \frac{1}{2}\begin{bmatrix}(1-\theta_w)\,\varepsilon_w(1+\varphi)\left(\varepsilon_w(1+\varphi)+1\right)\left(\Pi_t^{w*}-\Pi^{w*}\right)^2\\ +\varepsilon_w(1+\varphi)\left(\varepsilon_w(1+\varphi)-1\right)\theta_w(\Pi_t^w-\Pi^w)^2\end{bmatrix}\\
&+ \varepsilon_w(1+\varphi)\,\theta_w\left(\Pi_t^w-\Pi^w\right)\left(X_{t-1}^W - X^w\right)\ .
\end{aligned} \tag{D.51}$$

We already know that

$$\theta_w\left(\Pi_t^w-\Pi^w\right) - (1-\theta_w)\left(\Pi_t^{w*}-\Pi^{w*}\right) = \frac{1}{2}\left[(1-\theta_w)(-\varepsilon_w)\left(\Pi_t^{w*}-\Pi^{w*}\right)^2 - \theta_w(\varepsilon_w-2)\left(\Pi_t^w-\Pi^w\right)^2\right], \tag{D.24}$$

so that

$$\begin{aligned}
\hat{X}_t^W =& \frac{1}{2}\varepsilon_w(1+\varphi)\left[(1-\theta_w)(-\varepsilon_w)\left(\Pi_t^{w*}-\Pi^{w*}\right)^2 - \theta_w(\varepsilon_w-2)\left(\Pi_t^w-\Pi^w\right)^2\right]\\
&+ \theta_w\left(X_{t-1}^W - X^w\right)\\
&+ \frac{1}{2}\begin{bmatrix}(1-\theta_w)\,\varepsilon_w(1+\varphi)\left(\varepsilon_w(1+\varphi)+1\right)\left(\Pi_t^{w*}-\Pi^{w*}\right)^2\\ +\varepsilon_w(1+\varphi)\left(\varepsilon_w(1+\varphi)-1\right)\theta_w(\Pi_t^w-\Pi^w)^2\end{bmatrix}\\
&+ \varepsilon_w(1+\varphi)\,\theta_w\left(\Pi_t^w-\Pi^w\right)\left(X_{t-1}^W - X^w\right)\ .
\end{aligned} \tag{D.52}$$

Like S_t^W, the dispersion related term $\hat{X}_t^W$ is zero up to first order. For that reason, we can immediately drop the last term in the previous equation as being third order. Using

$$\left(\frac{\theta_w}{(1-\theta_w)}\right)^2 (\Pi_t^w - \Pi^w)^2 = (\Pi_t^{w*} - \Pi^{w*})^2 \tag{D.22}$$

we obtain:

$$\begin{aligned}
\hat{X}_t^W =& \frac{1}{2}\varepsilon_w (1+\varphi) \begin{bmatrix} (1-\theta_w)(-\varepsilon_w)\left(\frac{\theta_w}{(1-\theta_w)}\right)^2 \left(\hat{\Pi}_t^w\right)^2 \\ -\theta_w (\varepsilon_w - 2)\left(\hat{\Pi}_t^w\right)^2 \end{bmatrix} + \theta_w \left(X_{t-1}^W - X^w\right) \\
&+ \frac{1}{2} \begin{bmatrix} (1-\theta_w)\varepsilon_w (1+\varphi)(\varepsilon_w(1+\varphi)+1)\left(\frac{\theta_w}{(1-\theta_w)}\right)^2 \left(\hat{\Pi}_t^w\right)^2 \\ +\varepsilon_w (1+\varphi)(\varepsilon_w(1+\varphi)-1)\theta_w \left(\hat{\Pi}_t^w\right)^2 \end{bmatrix} \\
=& \theta_w \left(X_{t-1}^W - X^w\right) + \frac{1}{2}(1+\varphi) \begin{bmatrix} (1-\theta_w)\left(\frac{\theta_w}{(1-\theta_w)}\right)^2 \left(\hat{\Pi}_t^w\right)^2 \left(-\varepsilon_w^2 + \varepsilon_w^2 (1+\varphi) + \varepsilon_w\right) \\ -\theta_w \left(\hat{\Pi}_t^w\right)^2 \left(-\varepsilon_w^2 - 2\varepsilon_w + \varepsilon_w^2(1+\varphi) - \varepsilon_w\right) \end{bmatrix} \\
=& \theta_w \left(X_{t-1}^W - X^w\right) + \frac{1}{2}(1+\varphi) \begin{bmatrix} \frac{\theta_w^2}{(1-\theta_w)}\left(\hat{\Pi}_t^w\right)^2 (\varepsilon_w(1+\varphi\varepsilon_w)) \\ +\theta_w \left(\hat{\Pi}_t^w\right)^2 (\varepsilon_w(1+\varphi\varepsilon_w)) \end{bmatrix} \\
=& \theta_w \left(X_{t-1}^W - X^w\right) + \frac{1}{2}(\varepsilon_w(1+\varphi\varepsilon_w))\theta_w(1+\varphi)\left[\frac{\theta_w}{(1-\theta_w)} + 1\right]\left(\hat{\Pi}_t^w\right)^2 \\
=& \theta_w \left(X_{t-1}^W - X^w\right) + \frac{1}{2}(\varepsilon_w(1+\varphi\varepsilon_w))\theta_w(1+\varphi)\left[\frac{\theta_w}{(1-\theta_w)} + \frac{(1-\theta_w)}{(1-\theta_w)}\right]\left(\hat{\Pi}_t^w\right)^2 \\
=& \theta_w \left(X_{t-1}^W - X^w\right) + \frac{1}{2}(1+\varphi)\frac{(\varepsilon_w(1+\varphi\varepsilon_w))\theta_w}{1-\theta_w}\left(\hat{\Pi}_t^w\right)^2
\end{aligned} \tag{D.53}$$

Analogous to (D.28), the discounted sum of the auxiliary price dispersion term is given by

$$\sum_{t=t_0}^{\infty} \beta^{t-t_0} \hat{X}_t^W = \frac{\theta_w}{1-\beta\theta_w}\hat{X}_{t_0-1}^W + \sum_{t=t_0}^{\infty}\beta^{t-t_0}\frac{1}{2}\left(\frac{(1+\varphi)(\varepsilon_w(1+\varphi\varepsilon_w))\theta_w}{(1-\beta\theta_w)(1-\theta_w)}\left(\hat{\Pi}_t^w\right)^2\right). \tag{D.54}$$

The present discounted welfare loss then follows as:

$$\begin{aligned}
\sum_{t=t_0}^{\infty} \beta^{t-t_0}\Delta U^{EHL} =& \frac{N^{1+\varphi}}{(1-\alpha)}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left(\frac{\phi_w^{EHL}}{2}\left(\hat{\Pi}_t^w\right)^2 - \frac{(1-\alpha)}{1+\varphi}\hat{X}_t^W\right) \\
=& \frac{N^{1+\varphi}}{(1-\alpha)}\left[-\frac{\theta_w}{1-\beta\theta_w}\hat{X}_{t_0-1}^W + \sum_{t=t_0}^{\infty}\beta^{t-t_0}\frac{1}{2}\left(\phi_w^{EHL} - \frac{(1-\alpha)}{1+\varphi}\frac{(1+\varphi)(\varepsilon_w(1+\varphi\varepsilon_w))\theta_w}{(1-\beta\theta_w)(1-\theta_w)}\right)\left(\hat{\Pi}_t^w\right)^2\right].
\end{aligned} \tag{D.55}$$

Conditional on initial wage dispersion being 0 so that $\hat{X}_{t_0-1}^W = 0$, we get the conditional welfare difference as:

$$
\begin{aligned}
&E_t \sum_{t=t_0}^{\infty} \beta^{t-t_0} \Delta U^{EHL} \\
&= \frac{N^{1+\varphi}}{(1-\alpha)} E_t \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left(\frac{\phi_w^{EHL}}{2} \left(\hat{\Pi}_t^w\right)^2 - \frac{(1-\alpha)}{1+\varphi} \hat{X}_t^W \right) \\
&= \frac{N^{1+\varphi}}{(1-\alpha)} \left[\sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{1}{2} \left(\phi_w^{EHL} - \frac{(1-\alpha)}{1+\varphi} \frac{(1+\varphi)\left(\varepsilon_w \left(1+\varphi\varepsilon_w\right)\right)\theta_w}{\left(1-\beta\theta_w\right)\left(1-\theta_w\right)} \right) E_t \left(\hat{\Pi}_t^w\right)^2 \right] \\
&= \frac{N^{1+\varphi}}{(1-\alpha)} \left[\sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{1}{2} \left(\phi_w^{EHL} - \frac{(1-\alpha)\left(\varepsilon_w \left(1+\varphi\varepsilon_w\right)\right)\theta_w}{\left(1-\beta\theta_w\right)\left(1-\theta_w\right)} \right) E_t \left(\hat{\Pi}_t^w\right)^2 \right] \\
&= N^{1+\varphi} \left[\sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{1}{2} \left(\frac{\left(\varepsilon_w - 1\right)\theta_w \left(1-\tau^n\right) \aleph \left(1+\varphi\varepsilon_w\right)}{(1-\alpha)\left(1-\theta_w\right)\left(1-\beta\theta_w\right)} - \frac{\varepsilon_w \left(1+\varphi\varepsilon_w\right)\theta_w}{\left(1-\beta\theta_w\right)\left(1-\theta_w\right)} \right) E_t \left(\hat{\Pi}_t^w\right)^2 \right] ,
\end{aligned}
\tag{D.56}
$$

where the last line imposes identical slopes of the wage PC via:

$$
\phi_w^{EHL} = \frac{\left(\varepsilon_w - 1\right)\theta_w \left(1-\tau^n\right) \aleph \left(1+\varphi\varepsilon_w\right)}{\left(1-\theta_w\right)\left(1-\beta\theta_w\right)} . \tag{D.57}
$$

In an undistorted steady state with $\aleph = (1-\alpha)$ and $(1-\tau^n) = \frac{\varepsilon_w}{(\varepsilon_w - 1)}$:

$$
E_t \sum_{t=t_0}^{\infty} \beta^{t-t_0} \Delta U^{EHL} = N^{1+\varphi} \left\{ \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{1}{2} \left(\frac{\varepsilon_w \theta_w \left(1+\varphi\varepsilon_w\right)}{\left(1-\theta_w\right)\left(1-\beta\theta_w\right)} - \frac{\varepsilon_w \theta_w \left(1+\varphi\varepsilon_w\right)}{\left(1-\beta\theta_w\right)\left(1-\theta_w\right)} \right) E_t \left(\hat{\Pi}_t^w\right)^2 \right\} = 0. \tag{D.58}
$$

This completes the proof of Proposition 2 for the EHL framework.

Finally, consider unconditional welfare. Taking the unconditional expectations of equation (D.53) we get

$$
E\hat{X}_t^W = \theta_w \hat{X}_{t-1}^W + \frac{1}{2}(1+\varphi) \frac{\left(\varepsilon_w \left(1+\varphi\varepsilon_w\right)\right)\theta_w}{1-\theta_w} E\left(\hat{\Pi}_t^w\right)^2 \tag{D.59}
$$

so that the unconditional mean of the auxiliary price dispersion term is given by:

$$
E\hat{X}_t^W = \frac{1}{2}(1+\varphi) \frac{\left(\varepsilon_w \left(1+\varphi\varepsilon_w\right)\right)\theta_w}{\left(1-\theta_w\right)^2} E\left(\hat{\Pi}_t^w\right)^2 . \tag{D.60}
$$

Taking the unconditional expectations in (D.55) and using the previous result, we obtain:

$$
\begin{aligned}
&E\sum_{t=t_0}^{\infty}\beta^{t-t_0}\Delta U^{EHL} \\
&=\frac{N^{1+\varphi}}{(1-\alpha)}\left[\sum_{t=t_0}^{\infty}\beta^{t-t_0}\frac{1}{2}\left(\phi_w^{EHL}-(1-\alpha)\frac{(\varepsilon_w(1+\varphi\varepsilon_w))\theta_w}{(1-\beta\theta_w)(1-\theta_w)}\right)E\Big(\hat{\Pi}_t^w\Big)^2-\frac{(1-\alpha)}{1+\varphi}\frac{\theta_w}{1-\beta\theta_w}E\hat{X}_{t_0-1}^W\right] \\
&=\frac{N^{1+\varphi}}{(1-\alpha)}\begin{bmatrix}\frac{1}{2}\left(\phi_w^{EHL}-(1-\alpha)\frac{(\varepsilon_w(1+\varphi\varepsilon_w))\theta_w}{(1-\beta\theta_w)(1-\theta_w)}\right)E\Big(\hat{\Pi}_t^w\Big)^2\frac{1}{1-\beta} \\ -\frac{(1-\alpha)}{1+\varphi}\frac{\theta_w}{1-\beta\theta_w}\frac{1}{2}(1+\varphi)\frac{(\varepsilon_w(1+\varphi\varepsilon_w))\theta_w}{(1-\theta_w)^2}E\Big(\hat{\Pi}_t^w\Big)^2\end{bmatrix} \\
&=\frac{1}{2}\frac{N^{1+\varphi}}{(1-\alpha)}\left[\phi_w^{EHL}-(1-\alpha)\frac{(\varepsilon_w(1+\varphi\varepsilon_w))\theta_w}{(1-\beta\theta_w)(1-\theta_w)}-(1-\beta)(1-\alpha)\frac{\theta_w}{1-\beta\theta_w}\frac{(\varepsilon_w(1+\varphi\varepsilon_w))\theta_w}{(1-\theta_w)^2}\right].
\end{aligned}
\tag{D.61}
$$

Again imposing identical slopes of the wage Phillips curves, we get:

$$
E\sum_{t=t_0}^{\infty}\beta^{t-t_0}\Delta W^{EHL}=\frac{1}{2}\frac{N^{1+\varphi}}{(1-\alpha)}\phi_w^{EHL}\begin{pmatrix}1-\frac{(1-\alpha)\varepsilon_w}{(\varepsilon_w-1)\aleph(1-\tau^n)} \\ -\frac{(1-\beta)(1-\alpha)\theta_w}{(1-\theta_w)}\frac{(1-\alpha)\varepsilon_w}{(\varepsilon_w-1)\aleph(1-\tau^n)}\end{pmatrix}\frac{1}{1-\beta}E\Big(\hat{\Pi}_t^w\Big)^2
\tag{D.62}
$$

In an undistorted steady state with $\aleph=(1-\alpha)$ and $(1-\tau^n)=\frac{\varepsilon_w}{(\varepsilon_w-1)}$:

$$
\begin{aligned}
E\sum_{t=t_0}^{\infty}\beta^{t-t_0}\Delta U^{EHL}&=\frac{1}{2}\frac{N^{1+\varphi}}{(1-\alpha)}\phi_w^{EHL}\begin{pmatrix}1-\frac{(1-\alpha)\varepsilon_w}{(\varepsilon_w-1)\aleph(1-\tau^n)} \\ -\frac{(1-\beta)(1-\alpha)\theta_w}{(1-\theta_w)}\frac{(1-\alpha)\varepsilon_w}{(\varepsilon_w-1)\aleph(1-\tau^n)}\end{pmatrix}\frac{1}{1-\beta}E\Big(\hat{\Pi}_t^w\Big)^2 \\
&=\frac{1}{2}\frac{N^{1+\varphi}}{(1-\alpha)}\phi_w^{EHL}\left(-\frac{(1-\beta)(1-\alpha)\theta_w}{(1-\theta_w)}\right)\frac{1}{1-\beta}E\Big(\hat{\Pi}_t^w\Big)^2 \\
&=\frac{1}{2}N^{1+\varphi}\phi_w^{EHL}\left(-\frac{(1-\beta)\theta_w}{(1-\theta_w)}\right)\frac{1}{1-\beta}E\Big(\hat{\Pi}_t^w\Big)^2.
\end{aligned}
\tag{D.63}
$$

Again using (D.40) it follows that:

$$
E\sum_{t=t_0}^{\infty}\beta^{t-t_0}\Delta U^{EHL}=\frac{1}{2}N^{1+\varphi}\phi_w^{EHL}\left(1-\frac{1-\beta\theta_w}{1-\theta_w}\right)\frac{1}{1-\beta}E\Big(\hat{\Pi}_t^w\Big)^2<0 \tag{D.64}
$$

Thus, Calvo wage setting is associated with higher unconditional welfare losses. This completes the proof of Proposition 3 for the SGU case.

D.4 Comparison SGU/EHL with efficient steady state

The previous sections have compared Calvo and Rotemberg pricing in the SGU and EHL setup, respectively, for a given amount for wage stickiness as measured by the Calvo wage setting parameter. But for a given Calvo wage setting parameter, the EHL and SGU setups produce very different slopes of the wage Phillips Curve, implying that amount of wage inflation variability will differ across the two insurance scheme. Thus, to make the EHL and SGU frameworks comparable, we need to fix the slope of the Wage Phillips Curve at the same level by setting the respective Calvo parameters to satisfy

$$\phi_w = \phi_w^{EHL} = \frac{(\varepsilon_w - 1)\,\theta_w^{EHL}\,(1-\tau^n)\,\aleph\,(1+\varphi\varepsilon_w)}{(1-\theta_w^{EHL})\,(1-\beta\theta_w^{EHL})} = \frac{(\varepsilon_w - 1)\,\theta_w^{SGU}\,(1-\tau^n)\,\aleph}{(1-\theta_w^{SGU})\,(1-\beta\theta_w^{SGU})} = \phi_w^{SGU}\,. \tag{D.65}$$

This in turn implies that $\theta_w^{EHL} < \theta_w^{SGU}$. Put differently, the slope of the wage Phillips Curve under Calvo pricing is steeper in the EHL than in the SGU framework for a given amount of Calvo wage stickiness. In order to keep the slope of the Wage Phillips Curve the same, the EHL setup requires a lower degree of Calvo wage stickiness. The difference in unconditional welfare between the SGU and the EHL case under identical slopes of the Wage Phillips Curve then follows from (D.41) and (D.64) as

$$\begin{aligned}
&E\sum_{t=t_0}^{\infty}\beta^{t-t_0}\Delta U^{SGU} - E\sum_{t=t_0}^{\infty}\beta^{t-t_0}\Delta U^{EHL} \\
&= \frac{1}{2}N^{1+\varphi}\frac{\phi_w}{1-\beta}\left(1-\frac{1-\beta\theta_w^{SGU}}{1-\theta_w^{SGU}} - \left(1-\frac{1-\beta\theta_w^{EHL}}{1-\theta_w^{EHL}}\right)\right)E\left(\hat{\Pi}_t^w\right)^2 \\
&= \frac{1}{2}N^{1+\varphi}\frac{\phi_w}{1-\beta}\left(\frac{1-\beta\theta_w^{EHL}}{1-\theta_w^{EHL}} - \frac{1-\beta\theta_w^{SGU}}{1-\theta_w^{SGU}}\right)E\left(\hat{\Pi}_t^w\right)^2 < 0\,,
\end{aligned} \tag{D.66}$$

where we used that inflation variability is the same in both setups with $\phi_w = \phi_w^{EHL} = \phi_w^{SGU}$ and that $\theta_w^{EHL} < \theta_w^{SGU}$. This proves Proposition 4